GUDLAVALLERU ENGINEERING COLLEGE
(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)
Seshadri Rao Knowledge Village, Gudlavalleru – 521 356.

HANDOUT

on

FUZZY MATHEMATICS

OPEN ELECTIVE

Vision

To be a Centre of Excellence in computer science and engineering education and training to meet the challenging needs of the industry and society

Mission

- To impart quality education through well-designed curriculum in tune with the growing software needs of the industry.
- To be a Centre of Excellence in computer science and engineering education and training to meet the challenging needs of the industry and society.
- To serve our students by inculcating in them problem solving, leadership, teamwork skills and the value of commitment to quality, ethical behaviour & respect for others.
- To foster industry-academia relationship for mutual benefit and growth.

Program Educational Objectives

PEO1 : Identify, analyze, formulate and solve Computer Science and Engineering problems both independently and in a team environment by using the appropriate modern tools.

PEO2 : Manage software projects with significant technical, legal, ethical, social, environmental and economic considerations.

PEO3 : Demonstrate commitment and progress in lifelong learning, professional development, leadership and Communicate effectively with professional clients and the public.

HANDOUT ON FUZZY MATHEMATICS

Class & Sem. :III B.Tech – I Semester Year :2018-19

Branch :CSE Credits: 3

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1. Brief History and Scope of the Subject:

Only in twentieth century, mathematics defined the concepts of sets and functions to represent problems. This way of representing problems is more rigid. This difficulty was overcome by the fuzzy concept. In all Engineering, Medicine and other fields etc. concepts have been redefined using fuzzy sets. Hence it is a must to popularize these ideas for our future generation. In order to study the control problems of complicate systems and dealing with fuzzy information, American Cyberneticist L. A. Zadeh introduced Fuzzy Set Theory in 1965, describing fuzziness mathematically for the first time. The first phase, from 1965 to 1973 was concerned with fuzzification. In the second phase, two key concepts were introduced, (1)the concept of a linguistic variable and (2)the concept of a fuzzy if-then rule. Then the term fuzzy logic was used for the first time in 1974.Today, almost all applications of fuzzy set theory and fuzzy logic involve the use of these concepts. An important development in the evolution of fuzzy logic, marking the beginning of the third phase, is the genesis of computing with words and the computational theory of perceptions. Fuzzy logic has been successfully used in numerous fields such as control systems engineering, image processing, power engineering, industrial automation, robotics, consumer electronics, and optimization. This branch of mathematics has instilled new life into scientific fields that have been dormant for a long time.

The number of research contributions is growing daily and is growing at an increasing rate. Zadeh started the Berkeley Initiative in Soft Computing (BISC), a famous research laboratory at University of California, Berkeley, to advance theory and applications of fuzzy logic and soft computing.

2. Pre-Requisites:

- Mathematics background such as set theory and relations, and the fundamentals of Group Theory and Ring Theory.

3. Course Objectives:

- To know the fundamentals of fuzzy algebra
- To know the basic definitions of fuzzy theory
- To know the applications of fuzzy Technology

4.Course Outcomes:

Students should be able to:

CO1 understand the fundamentals of fuzzy algebra
CO2: apply fuzzy logic

5.Program Outcomes:

Graduates of the Computer Science and Engineering Program will have

a) an ability to apply knowledge of mathematics, science, and engineering
b) an ability to design and conduct experiments, as well as to analyze and interpret data
c) an ability to design a system, component, or process to meet desired needs within realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability
d) an ability to function on multidisciplinary teams
e) an ability to identify, formulate, and solve engineering problems
f) an understanding of professional and ethical responsibility
g) an ability to communicate effectively
h) the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context
i) a recognition of the need for, and an ability to engage in life-long learning,
j) a knowledge of contemporary issues
k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.

6.Mapping of Course Outcomes with Program Outcomes:

	a	b	c	d	e	f	g	h	i	j	k
CO1	H		M	M	M			M			
CO2			H	H	H			H			

H:High M:Medium

7.Prescribed Text Books:

a. S.Nanda and N.R.Das “Fuzzy Mathematical concepts, Narosa Publishing House, New Delhi.

8.Reference Text Books:

a. Fuzzy Logic with Engineering Applications, Second Edition, Wiley Publications, Timothy J.Ross.
b. Fuzzy Set Theory and Its Applications, Fourth Edition,Yes Dee Publishing Pvt. Ltd., Springer, H.-J. Zimmermann.

9.URLs and Other E-Learning Resources

a. *fuzzy sets:* https://www.calvin.edu/.../**Fuzzy**/**fuzzysets**.htm

b. *fuzzy relations:*
*osp.mans.edu.eg/elbeltagi/AI%20**FuzzyRelations**.pdf*

c. *fuzzy logic:*
http://www.seattlerobotics.org/encoder/mar98/fuz/flindex.html

10. Digital Learning Materials:

https://www.**youtube**.com/watch?v=H9SikB7HbSU
https://www.**youtube**.com/watch?v=n9eNXs76VVM

11. Lecture Schedule / Lesson Plan:

Topic	No. of Periods	
	Theory	Tutorial
UNIT –1:		
Introduction	2	
Fuzzy subsets and its properties	4	
Partially ordered sets	2	
Lattice and Boolean Algebras	4	
L fuzzy sets	2	
UNIT – 2:		
operations on fuzzy sets	2	
Disjunctive sum, α levels sets	2	
properties of fuzzy subsets of a set	2	
UNIT – 3:		
Algebraic product and sum of two fuzzy subsets	2	
properties satisfied by addition and product	3	
Cartesian product of fuzzy subsets	2	
UNIT – 4:		
Fuzzy Relations	3	
Algebra of fuzzy relations	3	
Properties of fuzzy relations	2	
Logic	2	
Connectives	3	
UNIT – 5:		
Connectives – Exclusive or, NAND, NOR	2	
Fuzzy Logic	2	
fuzzy subgroup & Lattice of Fuzzy Sub Group	3	
homomorphic image and Pre- image of subgroupoid	3	
UNIT – 6:		
Fuzzy invariant subgroups	3	
fuzzy subrings	3	
Total No.of Periods:	**56**	

UNIT – I

Objective:

The objective of this course is to

- Provide an emphasis on the differences and similarities between fuzzy set and crisp set fundamentals.
- Become familiar with posets, lattice, boolean algebra and with special properties of membership function such as support, normal etc.

Syllabus: Introduction, fuzzy subsets, posets, lattice, Boolean algebra, L- fuzzy sets

Learning Outcomes:

At the end of the unit students should be able to

- Distinguish between the crisp set and fuzzy set concepts.
- Develop fuzzy set using linguistic words and represent these sets by membership functions.
- List out fuzzy set related properties such as support, normal, subnormal etc..

Learning Material

FUZZY SUBSETS & ITS PROPERTIES :

Fuzzy Subset:

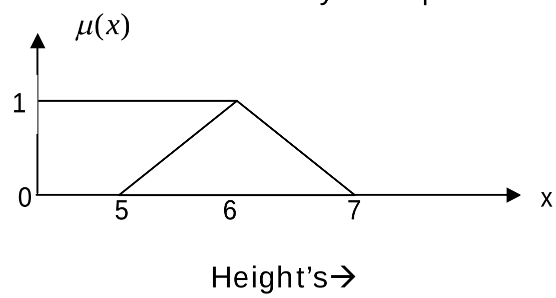
Let X be a set and x be an arbitrary element of X. Then a fuzzy subset of X is a map μ: X → [0,1] or is a set of ordered pairs {(x, μ(x)) / x$\in$X }where μ(x) is the degree of membership of x in X which take values in [0,1] and μ is called membership function on X.

(OR)

A fuzzy subset is defined by a membership function. A membership function assigns to each element in the set under consideration (universe or universal space or universe of discourse)a membership grade which is a value in[0,1].By defining a set using membership function it is possible for an element to belong partially to a set.

Ex:-

1) Let X = {g_1, g_2, g_3,g_4,g_5} be the set of students. Let A be the fuzzy set or fuzzy subset of smart students where 'smart' is the fuzzy linguistic variable and is given by
   A = {(g_1, .4), (g_2, .5), (g_3, 1), (g_4, .9) (g_5, .8)}
2) A = { height's around 6 feet}
   The set of height's around 6 feet is fuzzy or imprecise.



Height membership function for a fuzz set A.

PROPERTIES :

<u>Support of a fuzzy subset</u>:- If μ is a fuzzy subset then support of μ = {$x \in X$ / μ (x) >0}.

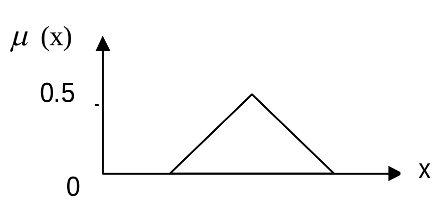
(OR)

The support of a fuzzysubset comprises those elements of the universe whose membership grade is greater than 0.

<u>Height of a fuzzy subset</u>:- If μ is a fuzzy subset of X then height of μ is denoted by hgt(μ)and is defined as
hgt(μ)= sup {μ (x) / x $\in$X} i.e., Maximum value of the membership function.

<u>Normal</u>:- If μ is a fuzzy subset of X then u is said to be normal if height of μ = 1, other wise μ called a subnormal fuzzy subset.

Ex:-



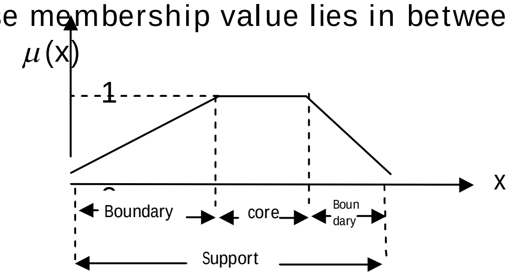
Sub normal Fuzzy subset with height = 0.5

<u>Cross over points of a fuzzy subset</u>:- If μ is a fuzzy subset of X then cross over points of $\mu = \left\{ x \in X / \mu(x) = \frac{1}{2} \right\}$

<u>Core:</u> The core consists of those elements of the universe whose membership value is 1.

<u>Boundary:</u> The boundary consists of those elements of the universe whose membership value lies in between 0 and 1.

Ex:-



<u>NOTE:</u> The above membership function is an example for normal fuzzy subset

<u>Partially Ordered Set</u>:- Let X be any non-empty set. A relation R on X is said to be a partial order relation on X if R is (1) Reflexive (2) Anti – Symmetric & (3) Transitive . A non Empty set together with partial order relation on X is called a poset or partially ordered setand is denoted by(X,R).

<u>Example</u>:- (1) (X,≤) is a poset where X is the set of all +ve integers and '≤' is a partial order relation on X.

Since $\forall x \in X,\ x \le x \Rightarrow '\le'$ is Reflexive

$\forall x, y \in X, x \le y \ \&\ y \le x \Rightarrow x = y \Rightarrow '\le'$ is Anti symmetric

$\forall x, y, z \in X, x \le y \ \&\ y \le z \Rightarrow x \le z \Rightarrow '\le'$ is transitive

(2) (X, $\subseteq$) is a poset where X is a family of all subsets of some universal set U and $\subseteq$ (inclusion) is a partial order relation on X.

Since $\forall A \in X, A \subseteq A \Rightarrow '\subseteq'$ is reflexive

$\forall A, B \in X, A \subseteq B \ \&\ B \subseteq A \Rightarrow A = B \Rightarrow '\subseteq'$ is Anti- symmetric

$\forall A, B, \subset \in X, A \subseteq B \ \&\ B \subseteq C \Rightarrow A \subseteq C \Rightarrow 'C'$ is Transitive

<u>Comparable:-</u> Two elements x and y in a poset are called comparable if either $x \le y$ or $y \le x$.

Ex :- (1) $(z^+, \le)$ is a poset , Here $\le$ means divides

2 & 4 are comparable since 2 $\le$ 4 i.e., 2/4

But 3 & 7 are not comparable since either $3 \nleq 7$ or $7 \nleq 3$.

(2) $Let X = \{1,2\}, P(X) = \{\phi, \{1\}, \{2\}, \{1,2\} = X\}$ is a power set of X then $(P(X), \subseteq)$ is a poset

Here $\phi, \{1\}$ ae comparable since $\phi \subseteq \{1\}$

But {1} ,{2} are not comparable since either {1} $\not\subseteq$ {2} or {2} $\not\subseteq$ {1}

Totally Order set or Linearly ordered set or a chain:- If any two elements are comparable in a poset then that poset is called totally ordered set.

Ex:- $(R, \leq)$ is a chain where R is the set of Real numbers

Supremum (or) Least upper bound (LUB) :- Let $(X, \leq)$ be a poset and Let A$\subseteq$X. An element $x \in X$ is said to be an upper bound for A if $a \leq x, \forall a \in A$. x is said to be supremum of A if x is an upper bound for A and if y is any other upper bound of A then $x \leq y$.

Infimum or greatest Lower bound(GLB):- Let $(X, \leq)$ be a poset and let A $\subseteq X$. An element x $\in$X is said to be a lower bound for A if x $\leq$ a , $\forall a \in A$. x is said to be infimum ofA if x is a lower bound for A if y is any lower bound of A then y $\leq x$

Ex:- $(R, \leq)$ is a poset where R is the set of Real numbers.

Let A = {1,2} .The elements which are $\geq$ 2 in R are upper bounds of A of all those upper bounds 2 is the supremum of A.

The elements which are ≤ 1 in R are lower bounds of A, of all those lower bounds 1 is the infimum of A.

Lattice:- A Poset $(X, \leq)$ is said to be Lattice if every two element subset of X has both supremum and infimum.

Ex:- Consider a poset $(P(X), \subseteq)$ is a lattice since $\forall A, B \in P(X), LUB$ is given by A$\cup$B and GLB is given by A$\cap B$.

Distributive Lattice:- A lattice is said to be distributive if the following distributive laws hold

(1) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

(2) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$, $\forall x, y, z$ in lattice.

Ex:- Lattice $(P(X), \subseteq)$ is a distributive lattice.

Complemented Lattice:- _ A Lattice is said to be complemented if it contains distinct element 0 and 1 $\ni 0 \leq x \leq 1 \forall x$ and (2) if each element x has a complement x^1 with the property that x V x^1 = 1, x $\wedge x^1$ = 0.

Ex:- A Lattice $(P(X),\subseteq)$ is complemented since each set in P(X) has its complement in P(X).

Boolean Algebra (or) Boolean Lattice:- A complemented distributive lattice is called a Boolean algebra.

Ex:- $(P(X),\subseteq)$ is a Boolean Algebra.

L-fuzzy sets:- Let X be a set and L be a Lattice. An L- Fuzzy subset of X is characterized by a mapping $\mu : X \to L$ and thus it can be represented by $\{(x,\mu(x))/\, x \in X\}$

Inclusion:- Let $\leq$ be the order relation of the Lattice L and μ_1 *and* μ_2 be two L – Fuzzy subsets of X.

μ_1 is said to tbe included in μ_2 if $\mu_1(x) \leq \mu_2(x),\ \forall x \in X$ *and it is denoted by* $\mu_1 \subseteq \mu_2$

Comparable:- Two L-Fuzzy subsets μ_1 *and* μ_2 are comparable if (1) the respective values taken by the membership function in the Lattice L are comparable.

Equality of L-Fuzzy subsets:- L – Fuzzy subsets μ_1 *and* μ_2 of X are said to be equal if $\mu_1(x) = \mu_2(x)\,\forall x \in X.$

Intersection of L-Fuzzy subsets:- Intersection of L – fuzzy subsets μ_1 *and* μ_2 of X is defined by $(\mu_1 \wedge \mu_2)(x) = \mu_1(x) \wedge \mu_2(x)\quad \forall x \in X$ i.e., max{ $\mu_1(x), \mu_2(x)$ }

Union of L-Fuzzy subsets:- Union of L – fuzzy subsets μ_1 *and* μ_2 is define by $(\mu_1 \cup \mu_2)(x) = \mu_1(x) \vee \mu_2(x)\,\forall x \in X$, i.e., max{ $\mu_1(x), \mu_2(x)$ }

Complement of a Fuzzy subset:- The complement of a fuzzy subset μ is well defined only when the Lattice is complemented if $\mu : X \to L$ then its complement μ^C is defined by $\mu^C(x)\quad if\ \ \mu(x) \vee \mu^C(x) = 1\ \&\ \mu(x) \wedge \mu^C(x) = 0$

OR

Two Fuzzy subsets μ_1 *and* μ_2 are said to be complementary if $\mu_2(x) = 1 - \mu_1(x)$, $\forall x \in X$ and this is denoted by $\mu_2 = \mu_1^c\ or\ \mu_1 = \mu_2^c$

Ex: -Let X={ a, b, c, d, e } then
μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} and μ_2 = {(a,0.7), (b,0.5), (c,0.3), (d,0), (e,1)} are complementary to each other.

UNIT-I
Assignment-Cum-Tutorial Questions
Section - A

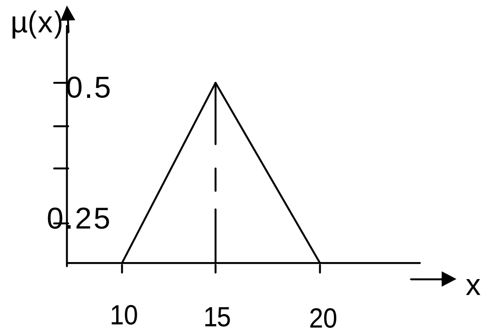
I) Objective Questions:

1. If μ_1 and μ_2 are two L-fuzzy subsets the ($\mu_1 \cup \mu_2$) (x) = -------------------------
2. If μ_1 and μ_2 are two L-fuzzy subsets the ($\mu_1 \cap \mu_2$) (x) = -------------------------
3. Two fuzzy subsets μ_1 and μ_2 are complementary if -----------------------------------
4. Let X={a, b, c, d, c, e} and μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} then μ_1^C = -----------------
5. Give an example of linguistic variable?
6. Define cross over points of a fuzzy subset?
7. Traditional set theory is also known as Crisp Set theory(True/False).----------------
8. Fuzzy logic is extension of Crisp logict with an extension of handling the concept of Partial Truth. (True/ False)------------------------.
9. For fuzzy sets where one and only one element has a membership equal to 1. This element is called as ----------------
10. Every totally ordered set is a --------------- [ ]
    a) Poset b) Lattice c) Boolean algebra d) None.
11. The fuzzy set associated with the membership function in Question no 8 is called ----------- [ ]
    a)Normal b) Subnormal c) Lattice d) None
12. Let '≤' be the order relation of the lattice L and μ_1 and μ_2 be two L-fuzzy subsets of X. μ_1 is said to be included in μ_2 if ------------------- [ ]
    a)$\mu_1(x) \geq \mu_2(x)\ \forall x$ □ X b) $\mu_1(x) = \mu_2(x)\ \forall\ x$ □ X
    c) $\mu_1(x) \leq \mu_2(x)\ \forall x$ □ X d) None
13. The set of perfect people is an example of ----------- [ ]
    a)Normal fuzzy set b) Sub normal fuzzy set c) Poset d) None
14. L-fuzzy subsets μ_1 and μ_2 of X are said to be equal if ---[ ]
    (a) $\mu_1(x) \neq \mu_2(x)$ v x □ X (b) $\mu_1(x_0) \neq \mu_2(x_0)$ for some x_0 □ X (c) $\mu_1(x) = \mu_2(x)$ v x □ X (d) None

Section - B

II) Multiple Choice Questions:

1. The height h(A) of a fuzzy set A is defined as h(A) =sup A(x) where x belongs to A. Then the fuzzyset A is called normal when[ ]
   a)h(A)=0 b)h(A)<0 c)h(A)=1 d) h(A)<1
2. is/are the way/s to represent uncertainty.[ ]
   a) Fuzzy Logic b) Probability
   c) Entropy d) All of the mentioned
3. The truth values of traditional set theory is ____________ and that of fuzzy set is __________ [ ]
   a) Either 0 or 1, between 0 & 1 b) Between 0 & 1, either 0 or 1
   c) Between 0 & 1, between 0 & 1 d) Either 0 or 1, either 0 or 1
4. The room temperature is hot. Here the hot (use of linguistic variable is used) can be represented by _______ . [ ]
   a) Fuzzy Set b) Crisp Set c)both d)none
5. The values of the set membership is represented by [ ]
   a)Discrete Set b) Degree of truth c) Probabilities d) Both b & c
6. Considering a graphical representation of the `tallness' of people using its appropriate member function, which of the following combinations are true ?[ ]
   i. TALL is usually the fuzzy subset.
   ii. HEIGHT is usually the fuzzy set.
   iii. PEOPLE is usually the universe of discourse.
   a)all I,ii and iii b)I and ii c) i d)none
7. If a relation R on a non-empty set X satisfies reflexive, anti-symmetric and transitive the R is called ----------- [ ]
   a)Linear order relation b) Partial order relation
   c) Both a and b d) None.
8. The core comprises those elements x of the universe for which μ(x) is ------ [ ]
   a)>1 b) <1 c) =1 d) > or =1
9. The support comprises those elements x of the universe for which μ(x) is -- [ ]
   a)>0 b) <0 c) =0 d) > or = 0
10. The boundary comprises those elements x of the universe whose membership value is --------- [ ]
    a)0< μ(x) < 1 b) 0.5 < μ(x) < 1 c) 0< μ(x) < 0.5 d) μ(x) = 1
11. A fuzzy set which is not normal is called as ---------------[ ]
    a)Sub normal b) L-fuzzy set c) Lattice d) None.
12. For the following membership function the height of a fuzzyset is -------- [ ]
    a)0 b) 0.5 c) 0.25 d) 1



Section – C

I)Problems:

1. Given X = [-1,1], $\mu_1(x) = |x|$; $\mu_2(x) = \begin{cases} 0 & \text{if } -1 \le x \le 0 \\ 1 - x & \text{if } 0 < x \le 1 \end{cases}$

   Sketch the graphical representation of μ_1 and μ_2.

2. In the above problem find support of μ_1 and support of μ_2. Are both μ_1 and μ_2 are normal ? Find the crossover points of μ_1 and μ_2.
3. Prepare a membership function for laminar and turbulent flow for a typical flat plate with a sharp leading edge in a typical air stream. Transition usually takes place between Reynolds numbers (Re) of $2x10^5$ and $3x10^6$. An Re of $5x10^5$ is usually considered the point of turbulent flow for this situation.
4. Sketch the graph of membership function for the following fuzzy sets.
   (a) Numbers approximately between 10 and 20.
   (b) Medium sized men.(c) High speed racing cars.

5. Develop a reasonable membership function for the following sets based on height measured in centimeters. a)Tall b) Short c) Not short.
6. Consider the set of people in the following age groups 0-10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70, 70 and above. Develop a reasonable membership function for the fuzzy set S.
   a)Young b) middle-aged c) old.
7. A Fuzzy set for a major storm event in a Calgary, Alberta could be described as a rainstorm in a subdivision that raised the level of the storm water pond to within 70% of its design capacity. The MF for a major storm set could be described as having full membership when 70% of the pond volume has been reached but varying from zero membership to full membership at 40% capacity and 70% capacity respectively. Draw typical membership function as it is described.
8. Model a reasonable membership function for a square based on geometric properties of a rectangle for this problem use L as the length of the longer side and *l* as the length of smaller side.

UNIT – II

Objectives: The objectives of this course are to

- Provide a brief introduction on operations and properties of a fuzzy subset.
- Become familiar with α-Level set.

Syllabus: operations on fuzzy subsets, Disjunctive sum, α-Level set, Properties of fuzzy subsets.

Learning Outcomes: At the end of the unit students will be able to

- Distinguish between crisp set operations and fuzzy set operations.
- Define α-cut of a fuzzy subset

Learning Material

Operations on fuzzy subsets:

- **Inclusion:-** μ_1 is said to be included in μ_2 or μ_1 is said to be a fuzzy subset of μ_2 if

$$\mu_1(x) \le \mu_2(x), \forall x \in X \text{ and it is denoted by } \mu_1 \subseteq \mu_2.$$

Example: Let μ_1 = {(a,0.3), (b,0.5), (c,0.3), (d,0), (e,0)} and μ_2 = {(a,0.7), (b,0.5), (c,0.7), (d,1), (e,1)} are two fuzzy sets clearly $\mu_1(x) \le \mu_2(x), \forall x \in X$.

- **Equality of Fuzzy subsets:-** Two Fuzzy subsets μ_1 *and* μ_2 of X are said to be equal if

$$\mu_1(x) = \mu_2(x) \forall x \in X.$$

Example: μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} and μ_2 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)}

- **Intersection of fuzzy subsets:-** Intersection of fuzzy subsets μ_1 *and* μ_2 of X is defined by

$$(\mu_1 \cap \mu_2)(x) = \min\{ \mu_1(x), \mu_2(x) \}$$

Example: Let μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} and μ_2 = {(a,0.7), (b,0.5), (c,0.3), (d,0), (e,1)} are two fuzzy sets then $\mu_1 \cap \mu_2$ = { (a,0.3), (b,0.5), (c,0.3), (d,0), (e,0)}

- **Union of Fuzzy subsets:-** Union of fuzzy subsets μ_1 *and* μ_2 of X is defined by

$$(\mu_1 \cup \mu_2)(x) = \max\{ \mu_1(x), \mu_2(x) \}$$

Example: Let μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} and μ_2 = {(a,0.7), (b,0.5), (c,0.3), (d,0), (e,1)} are two fuzzy sets then $\mu_1 \cup \mu_2$ = { (a,0.7), (b,0.5), (c,0.7), (d,1), (e,1)}

- **Complement of a Fuzzy subset:-** Two fuzzy subsets μ_1 *and* μ_2 are said to be complementary if

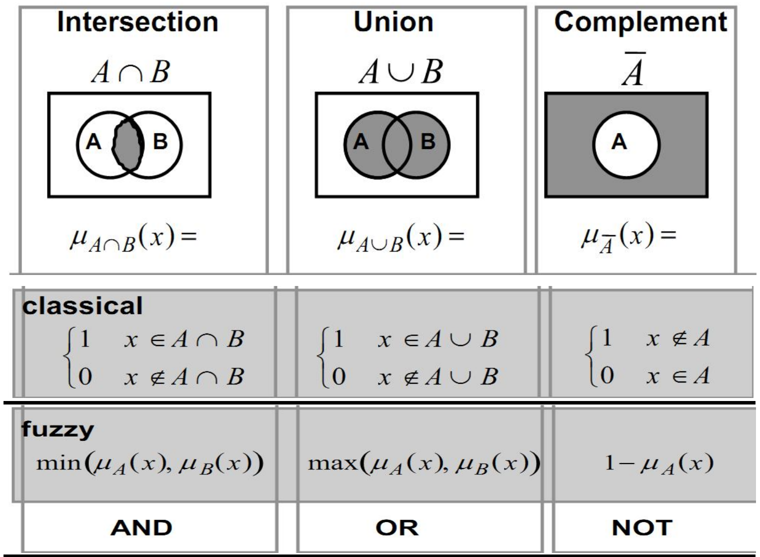
$$\mu_2(x) = 1 - \mu_1(x), \ \forall x \in X \text{ and this is denoted by } \mu_2 = \mu_1^c \text{ or } \mu_1 = \mu_2^c.$$

Ex: -Let X={ a, b, c, d, e } then

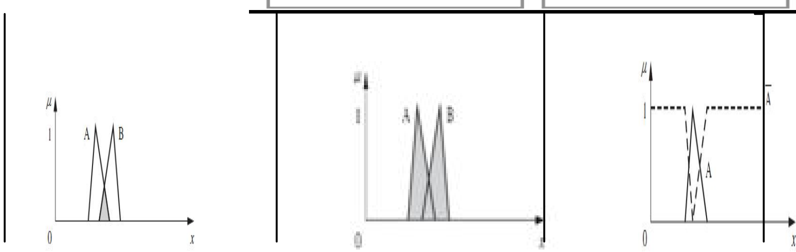
μ_1 = {(a,0.3), (b,0.5), (c,0.7), (d,1), (e,0)} and μ_2 = {(a,0.7), (b,0.5), (c,0.3), (d,0), (e,1)}are

complementary to each other.

NOTE: The following shows the operations on classical and fuzzy sets



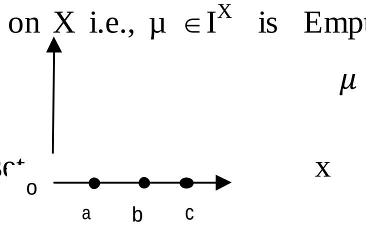
	Intersection	Union	Complement
	$A \cap B$	$A \cup B$	$\bar{A}$
	$\mu_{A \cap B}(x) =$	$\mu_{A \cup B}(x) =$	$\mu_{\bar{A}}(x) =$
classical	$\begin{cases} 1 & x \in A \cap B \\ 0 & x \notin A \cap B \end{cases}$	$\begin{cases} 1 & x \in A \cup B \\ 0 & x \notin A \cup B \end{cases}$	$\begin{cases} 1 & x \notin A \\ 0 & x \in A \end{cases}$
fuzzy	$\min(\mu_A(x), \mu_B(x))$	$\max(\mu_A(x), \mu_B(x))$	$1 - \mu_A(x)$
	AND	OR	NOT



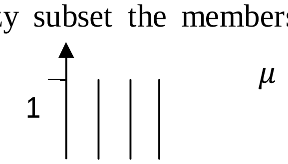
Definitions:→

Empty fuzzy subset: A fuzzy subset is called an Empty fuzzy subset which is denoted by 0 if its membership function is identically zero on X i.e., $\mu \in I^X$ is Empty fuzzy subset if μ (x) = 0 $\forall x \in X$

Example: μ = {(a,0), (b,0), (c,0)} is an empty fuzzy set



Universal fuzzy subset: Universal fuzzy subset is that fuzzy subset the membership function of which is identically 1 on X.



Example: $\mu = \{(a,1), (b,1), (c,1)\}$ is an universal fuzzy set

x

0 a b c

Note : $1^c = 0$ and $0^c = 1$

Disjoint fuzzy subsets: Two fuzzy subsets are said to be disjoint if their intersection is an empty fuzzy subset.

Ex: $\mu_1 = \{(a,.1),(b,.5),(c,0)\}$ and $\mu_2 = \{(a,0),(b,0),(c,.5)\}$ are disjoint

Since $\mu_1 \cap \mu_2 = \{(a,0),(b,0),(c,0)\}$

Note: 1) For an arbitrary collection $\{\mu_\alpha\}$ of fuzzy subsets of X the intersection $\bigcap_\alpha \mu_\alpha$ is defined by $\bigcap_\alpha \mu_\alpha = \inf_\alpha \mu_\alpha(x)$

2) $\bigcap_\alpha \mu_\alpha$ is the fuzzy subset contained in each μ_α

3) For an arbitrary collection $\{\mu_\alpha\}$ of fuzzy subsets of X the union is $\bigcup_\alpha \mu_\alpha$ is defined by

$$\bigcup_\alpha \mu_\alpha(x) = \sup_\alpha \mu_\alpha(x)$$

4) $\bigcup_\alpha \mu_\alpha$ is the fuzzy subset that contains each μ_α

Disjunctive sum: The disjunctive sum of two fuzzy subsets μ_1, μ_2 is denoted by $\mu_1 + \mu_2$ and is defined by $(\mu_1 + \mu_2)(x) = (\mu_1 \cap \mu_2^c) \cup (\mu_1^c \cap \mu_2) = \max\{(\mu_1 \cap \mu_2^c)(x), (\mu_1^c \cap \mu_2)(x)\}$

Difference: The difference of two fuzzy subsets μ_1 and μ_2 is denoted by $\mu_1 - \mu_2$ and is defined by $\mu_1 - \mu_2 = \mu_1 \cap \mu_2^c$

Note: $\mu_1 - \mu_2 \neq \mu_2 - \mu_1$

α-level set or α-cut: If μ is a fuzzy subset of X and $\alpha \in [0,1]$ the α-cut is denoted by α_μ or $[\mu]_\alpha$ is the crisp set that contains all the elements of the universal set whose membership grade in μ are $\geq \alpha$

Note: $[\mu]_{\alpha 1} \sqsubseteq [\mu]_{\alpha 2}$ if $\alpha 1 \geq \alpha 2$

Ex: Let A be a fuzzy set defined by $A = \{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$ $[A]_{.3} = \{4,5\}$

Properties of fuzzy subset of a set: If μ_1 ,μ_2, μ_3 are fuzzy subsets of a set X and 0,1 are respectively fuzzy null and fuzzy universal subsets then we have

1) $\mu_1 \cap \mu_2 = \mu_1 \cap \mu_2$
$\mu_1 \cup \mu_2 = \mu_1 \cup \mu_2$ } commutative

2) $\mu_1 \cup (\mu_2 \cup \mu_3) = (\mu_1 \cup \mu_2) \cup \mu_3$
$\mu_1 \cap (\mu_2 \cap \mu_3) = (\mu_1 \cap \mu_2) \cap \mu_3$ } Associative

3) $\mu_1 \cap \mu_1 = \mu_1$
$\mu_1 \cup \mu_1 = \mu_1$ } Idempotent

4) $(\mu_1 \cup \mu_2) \cap \mu_3 = (\mu_1 \cap \mu_3) \cup (\mu_2 \cap \mu_3)$
$(\mu_1 \cap \mu_2) \cup \mu_3 = (\mu_1 \cup \mu_3) \cap (\mu_2 \cup \mu_3)$ } Distributive

5) $(\mu_1^c)^c = \mu_1$ ___________ double complement

6) $(\mu_1 \cap \mu_2)^c = \mu_1^c \cup \mu_2^c$
$(\mu_1 \cup \mu_2)^c = \mu_1^c \cap \mu_2^c$ } Demorgan's law

7) $\mu_1 \cup 0 = \mu_1$
$\mu_1 \cap 1 = \mu_1$ } Identity

8) $\mu_1 \cup 1 = 1$ _______ Tautology

$\mu_1 \cap 0 = 0$ _______ Contradiction

Assignment-Cum-Tutorial Questions

A. Questions testing the remembering / understanding level of students

I) Objective Questions:

1. Empty fuzzy set is denoted by ___________

2. Universal fuzzy set is denoted by __________

3. $\mu \in I^X$ is empty fuzzy subset if ___________

4. The set of all fuzzy subsets on x is denoted by ___________

5. Two fuzzy sets μ_1 *and* μ_2 are said to be disjoint if ___________

6. For an arbitrary collection $\{\mu_\alpha\}$ of fuzzy sub sets of X, the union is $\cup \mu_\alpha$ defined by _______

7. For arbitrary collection $\{\mu_\alpha\}$ of fuzzy subsets of X the intersection $\cap \mu_\alpha$ is defined as ______

8. $(\mu_1 + \mu_2)(X)$ =________

9. $(\mu_1 - \mu_2)$ =s________

10. Is $(\mu_1 - \mu_2) \neq (\mu_2 - \mu_1)$? (Yes/No)

11. Define α-cut or α-Level set.

12. List out the properties of a fuzzy set.

II) Descriptive Questions:

1. When do we say two fuzzy subsets are disjoint, by giving an example?

2. State α-cut and give an example.

3. State the operations on a fuzzy set.

4. Define empty fuzzy set and disjoint fuzzy set.

5. Prove distributive law.

6. Prove Demorgan's law.

7. Does law of excluded middle holds good on fuzzy set? Justify your answer?

8. If μ and $\{\mu_i\}_{i\in I}$ are fuzzy subsets of X show that

(a) $\mu \cup (\bigcap_i \mu_i) = \bigcap_i (\mu \cup \mu_i)$

(b) $\left(\bigcup_i \mu_i\right)^C = \bigcap_i {\mu_i}^C$

9. The task is to recognize English alphabetical characters {F, E, X, Y, I, T} in an image processing system. Define two fuzzy sets A and B to represent the identification of characters I and F

Where A= {(F, 0.4), (E, 0.3), (X, 0.1), (Y, 0.1), (I, 0.9), (T, 0.8)} and

B = {(F, 0.99), (E, 0.8), (X, 0.1), (Y, 0.2), (I, 0.5), (T, 0.5)}

Find (a) (i) $A \cup B$ (ii) A-B (iii) $A \cup B^C$

(b) Verify Demorgan's law $(A \cup B)^C = A^C \cap B^C$

10. Let A be a fuzzy subset on X {-2, -1, 0, 1, 2, 3, 4} defined by A = $\frac{0}{-2}+\frac{0.3}{-1}+\frac{0.6}{0}+\frac{1}{1}+\frac{0.6}{2}+\frac{0.3}{3}+\frac{0}{4}$ list all α-cuts.

11. Show that the subsets μ_1 = {(a, 0.1), (b, 0.5), (c,0)} and μ_2 = {(a,0), (b,0), (c,0.5)} are disjoint.

12. What is the role of α-cut in fuzzy set theory?

B. Question testing the ability of students in applying the concepts.

I) Multiple Choice Questions:

Given two discrete fuzzy sets $A = \{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$ and $B = \{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$

To questions 1 to 9

1. $\bar{A}$ = __________ [ ]

a)$\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.4}{4} + \frac{0.8}{5}\}$ (b) $\{\frac{0}{2} + \frac{0.5}{3} + \frac{0.7}{4} + \frac{0.8}{5}$

(c) $\{\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}\}$ (d) None.

2. $A \cup B$ = _________ [ ]

(a) $\{\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}\}$ (b) $\{\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}\}$

(c) $\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}$ (d) None

3. $A \cap B$ = _________ [ ]

(a) $\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}$ (b) $\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}$ (c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (d) None

4. $(B|A) = B \cap \bar{A}$ = _________ [ ]

(a) $\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.3}{4} + \frac{0.2}{5}$ (b) $\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}$

(c) $\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.2}{5}$ (d) $\frac{0}{2} + \frac{0.5}{3} + \frac{0.2}{4} + \frac{0.4}{5}$

5. $A \cup B$ = _________ [ ]

(a) $\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5}$ (b) $\frac{1}{2} + \frac{0.7}{3} + \frac{0.3}{4} + \frac{0.4}{5}$

(c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}$ (d) None.

6.B^C = ______________ [ ]

(a) $\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}$ (b) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$

(c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (d) None.

7. $A^C \cup B^C$ = ________________ [ ]

(a $\frac{0.5}{2} + \frac{0.5}{3} + \frac{0.8}{4} + \frac{0.8}{5}$ (b) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}$ (d) None.

8. $A^C \cap B^C$ = ________________ [ ]

(a) $\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.8}{5}$ (b) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (d) None.

9. $A \cap B^C$ = ________________ [ ]

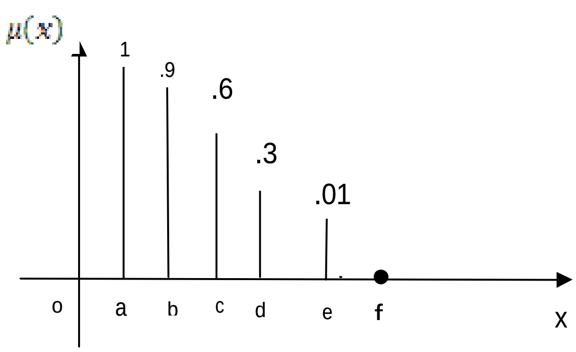
(a) $\frac{0.5}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (b) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.7}{4} + \frac{0.6}{5}$ (c) $\frac{0}{2} + \frac{0.3}{3} + \frac{0.8}{4} + \frac{0.6}{5}$ (d) None.

10. $A = \{\frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0}{5}\}$ is an example for _____________ [ ]

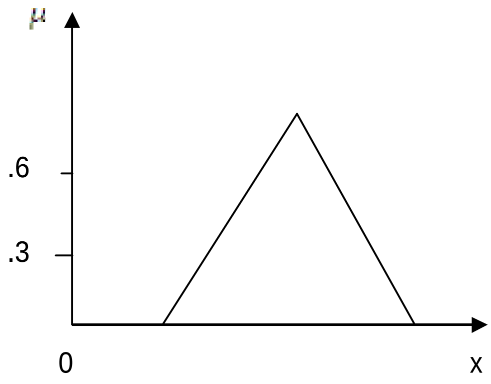
(a) empty fuzzy set (b) universal fuzzy set (c) both (a) and (b) (d) None.

II) Problems:

1) Reduce the following fuzzy set into several α-cut sets or list all α-cuts where the fuzzy set is given by $A = \frac{1}{a} + \frac{0.9}{b} + \frac{0.6}{c} + \frac{0.3}{d} + \frac{0.1}{e} + \frac{0}{f}$



2) From the following fuzzy set find α-cut when (i) $\alpha = 0.6$ graphically.



3) From question no.2) find α_cut *when* $\alpha = 0.3$

4) A four person family wants to buy a house. An indication of how comfortable they want to be is the number of bedrooms in the house. Let U = {1,2,-----,10} be the set of available houses described by their number of bed rooms. Then the fuzzy set C (for comfortable) may be described as

C = [0.2 0.5 0.8 1 0.7 0.3 0 0 0 0]. Let I be the fuzzy set large defined as I = [0 0 0.2 0.4 0.6 0.8 1 1 1 1]. Find (a) $C \cup I$ (b)$C \cap I$ (c) I^c

5) From question no.3 find i) $C^C \cup I$ ii) $(C^C)^C$

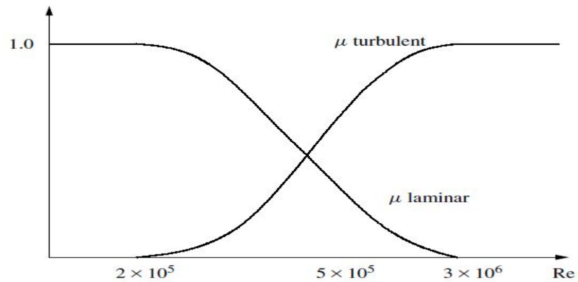
6) consider the fuzzy sets A and B defined on the interval X = [0, 5] of real numbers by the membership grade function. Given $\mu_A(x) = \frac{x}{x+1}$, $\mu_B(x) = 2^{-x}$. determine the mathematical formulae and graphs of the membership functions of each of the following sets.

(a) $A \cup B$ (b) $A \cap B$ (c) A^c, B^c (d) (a) $(A \cup B)^c$

C. Questions testing the Analyzing/Evaluating ability of students:

1) Typical membership functions for laminar and turbulent flow for a flat plate with a sharp leading edge in a typical air stream are shown in Figure P2.1. Transition between laminar and turbulent flow usually takes place between Reynolds numbers of 2 × 105 and 3 × 106. An *Re* = 5 × 105 is usually considered the point of turbulent flow for this situation. Find the

intersection, union, and the difference for the two flows. And, find the complement of laminar flow.

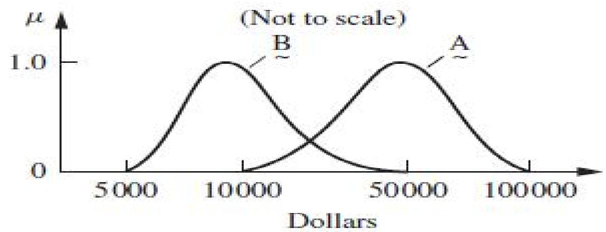


2) In neighborhoods there may be several storm-water ponds draining to a single downstream trunk sewer. In this neighborhood the city monitors all ponds for height of water caused by storm events. For two storms (labeled A and B) identified as being significant based on rainfall data collected at the airport, determine the corresponding performance of the neighborhood storm-water ponds. Suppose the neighborhood has five ponds, that is, X = [1, 2, 3, 4, 5], and suppose that significant pond storage membership is 1.0 for any pond that is 70% or more to full depth. For storm A, the pond performance set is A = $\left\{\frac{0.5}{1} + \frac{0.6}{2} + \frac{0.8}{3} + \frac{1}{4} + \frac{1}{5}\right\}$ For storm B , the pond performance set is B = $\{\frac{0.2}{1} + \frac{0.4}{2} + \frac{0.7}{3} + \frac{0.9}{4} + \frac{1}{5}\}$.

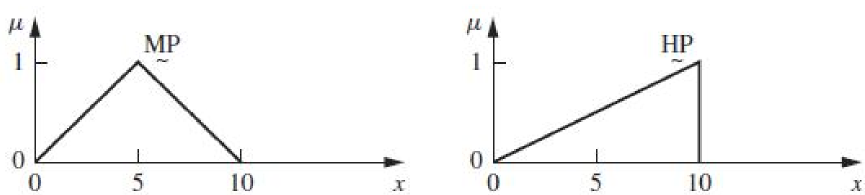
(a) To assess the impacts on pond performance suppose only two ponds can be monitored due to budget constraints. Moreover, data from the storms indicate that there may be a difference in thunder burst locations around this neighborhood. Which two of the five ponds should be monitored?

(b) Determine the most conservative estimate of pond performance (i.e., find A ∪B In determining corporate profitability, many construction companies must make decisions based upon the particular client's spending habits, such as the amount the client spends and their capacity for spending. Many of these attributes are fuzzy. A client which spends a "large amount" is considered to be "profitable" to the construction company. A "large" amount of spending is a fuzzy variable, as is a "profitable" return. These two fuzzy sets should have some overlap, but they should not be defined on an identical range.

(*a*) $\underset{\sim}{A} \cup \underset{\sim}{B}$: all clients deemed profitable or who are large spenders.
(*b*) $\underset{\sim}{A} \cap \underset{\sim}{B}$: all clients deemed profitable and large spenders.
(*c*) $\overline{\underset{\sim}{A}}$ and $\overline{\underset{\sim}{B}}$: those clients (i) deemed not profitable, and (ii) deemed not large spenders (separately).
(*d*) $\overline{\underset{\sim}{A}} \mid \overline{\underset{\sim}{B}}$: entities deemed profitable clients, but not large spenders.
(*e*) $\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$ (De Morgan's principle).



3) Suppose an engineer is addressing a problem in the power control of a mobile cellular telephone transmitting to its base station. Let MP ∼ be the medium-power fuzzy set and HP ∼ be the high-power set. Let the universe of discourse be composed of discrete units of d$B \times m$, that is, X = {0, 1, 2, . . . , 10}. The membership functions for these two fuzzy sets are shown in Figure P2.11. For these two fuzzy sets, demonstrate union, intersection, complement, and the difference.



Samples of a new microprocessor IC chip are to be sent to several customers for beta testing. The chips are sorted to meet certain maximum electrical characteristics, say frequency and temperature rating, so that the ‘‘best’’ chips are distributed to preferred customer 1. Suppose that each sample chip is screened and all chips are found to have a maximum operating frequency in the range 7–15 MHz at 20◦C. Also, the maximum operating temperature range (20◦C ±_T) at 8 MHz is determined. Suppose there are eight sample chips with the following

1. electrical characteristics:

	Chip number							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
f_{max}, MHz	6	7	8	9	10	11	12	13
ΔT_{max}, °C	0	0	20	40	30	50	40	60

$$\underset{\sim}{A} = \text{set of ``fast'' chips} = \text{chips with } f_{max} \geq 12 \text{ MHz}$$
$$= \left\{\frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0}{4} + \frac{0.2}{5} + \frac{0.6}{6} + \frac{1}{7} + \frac{1}{8}\right\}$$
$$\underset{\sim}{B} = \text{set of ``slow'' chips} = \text{chips with } f_{max} \geq 8 \text{ MHz}$$
$$= \left\{\frac{0.1}{1} + \frac{0.5}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right\}$$
$$\underset{\sim}{C} = \text{set of ``cold'' chips} = \text{chips with } \Delta T_{max} \geq 10^\circ\text{C}$$
$$= \left\{\frac{0}{1} + \frac{0}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right\}$$
$$\underset{\sim}{D} = \text{set of ``hot'' chips} = \text{chips with } \Delta T_{max} \geq 50^\circ\text{C}$$
$$= \left\{\frac{0}{1} + \frac{0}{2} + \frac{0}{3} + \frac{0.5}{4} + \frac{0.1}{5} + \frac{1}{6} + \frac{0.5}{7} + \frac{1}{8}\right\}$$

It is seen that the units for operating frequencies and temperatures are different; hence, the associated fuzzy sets could be considered from different universes and operations on combinations of them would involve the Cartesian product. However, both sets of universes have been transformed to a different universe, simply the universe of countable integers from 1 to 8. Based on a single universe, use these four fuzzy sets to illustrate various set operations. For example, the following operations relate the sets of ''fast'' and ''hot'' chips:

(*a*) $\underset{\sim}{A} \cup \underset{\sim}{D}$
(*b*) $\underset{\sim}{A} \cap \underset{\sim}{D}$
(*c*) $\overline{\underset{\sim}{A}}$
(*d*) $\underset{\sim}{A} \mid \underset{\sim}{D}$
(*e*) $\overline{\underset{\sim}{A} \cup \underset{\sim}{D}}$
(*f*) $\overline{\underset{\sim}{A} \cap \underset{\sim}{D}}$

4) Consider a local area network (LAN) of interconnected workstations that communicate using Ethernet protocols at a maximum rate of 10 Mbit/s. Traffic rates on the network can be expressed as the peak value of the total bandwidth (BW) used; and the two fuzzy variables, ''Quiet'' and ''Congested,'' can be used to describe the perceived loading of the LAN. If the discrete universal set X = {0, 1, 2, 5, 7, 9, 10} represents bandwidth usage, in Mbit/s, then the membership functions of the fuzzy sets Quiet $\underset{\sim}{Q}$ and Congested $\underset{\sim}{C}$ are as shown in Fig. P2.12.

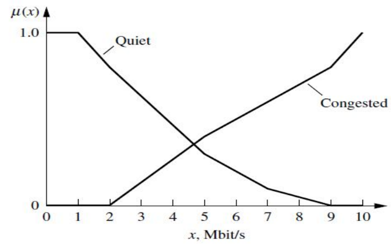
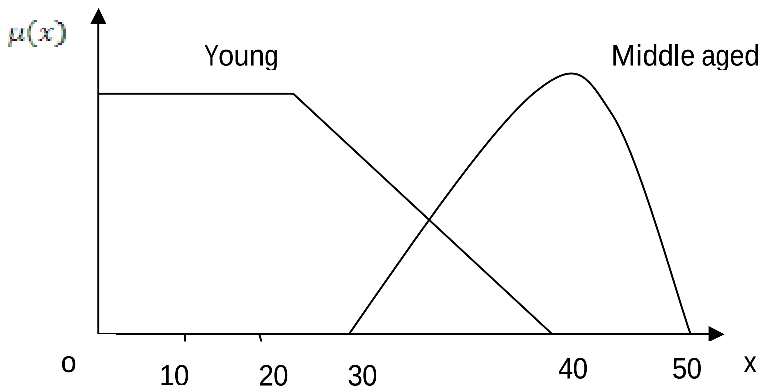


Fig P 2.12

For these two fuzzy sets, graphically determine the union, intersection, complement of each, difference $\underset{\sim}{Q} \mid \underset{\sim}{C}$, and both De Morgan's principles.

5) let A be the fuzzy set of young people and B be the fuzzy set of middle aged people as shown below.

Find (a) $A \cup B$ (b) $A \cap B$ (c) A^c



UNIT – III

Objective: The objective of this course is to

- Become familiar with Algebraic product and sum of two fuzzy subsets and properties satisfied by sum and product.

Syllabus: Algebraic Product and sum of two fuzzy subsets, properties satisfied by addition and product, Cartesian product of fuzzy subsets(definition).

Learning Outcomes: At the end of the unit students will be able to

- Understand algebraic product and sum
- List out properties satisfied by algebraic product and sum
- Define Cartesian product of fuzzy subsets.

Learning Material

Algebraic Product and sum of two fuzzy subsets :

- If μ_1, μ_2 are two fuzzy subsets of X then the product is denoted by $\mu_1 . \mu_2$ and sum is by $\mu_1 + \mu_2$ and is defined as $(\mu_1 . \mu_2)(x) = \mu_1(x)\, \mu_2(x)$ and $(\mu_1 + \mu_2)(x) = \mu_1(x) + \mu_2(x) - \mu_1(x)\, \mu_2(x)$.

Note: 1) $0 \le (\mu_1 + \mu_2)(x) \le 1$

2) $0 \le (\mu_1 . \mu_2)(x) \le 1$

Properties satisfied by sum and product:

If μ_1, μ_2 are two fuzzy subsets of X then

1) $\mu_1 . \mu_2 = \mu_2 . \mu_1$, $\mu_1 + \mu_2 = \mu_2 + \mu_1$ } commutative

2) $(\mu_1 + \mu_2) + \mu_3 = \mu_1 + (\mu_2 + \mu_3)$ } associative

$(\mu_1 . \mu_2) . \mu_3 = \mu_1 .(\mu_2 . \mu_3)$

3) $\mu . 0 = 0$, 0 being null fuzzy subset of X i.e.., $0(x) = 0 \ \forall x \in X$

$\mu + 0 = \mu$

4) $\mu .1 = \mu$,1 being universal fuzzy subset of X i.e.., $1(x)=1 \ \forall x \in X$

$\mu + 1 = 1$

5) $(\mu_1 . \mu_2)^c = \mu_1^c + \mu_2^c$

$(\mu_1 + \mu_2)^c = \mu_1^c . \mu_2^c$ } Demorgan's law

Note: Distributive laws are not satisfied under algebraic sum and product

Fuzzy subset function: Let $f : X \rightarrow Y$ be a function from a set X into Y and $\mu \in I^Y$. $f^{-1}(\mu)$ is a fuzzy subset of X defined by $f^{-1}(\mu)(x) = \mu(f(x)) \ \forall x$ □ X. If $g \in I^X$,f(g) is a fuzzy subset of Y defined by

$$f(g)(y) = \begin{cases} \sup\{g(x) / x \in f^{-1}(y) \neq \emptyset\} \\ 0 \quad \text{if } f^{-1} = \emptyset \end{cases}$$

NOTE: Let $f: X \rightarrow Y$ be a function then

1) $f^{-1}(g^c) = (f^{-1}(g))^c$ for any fuzzy set g of Y.

2) $f(g^c) \supset (f(g))^c$ for any subset g of X if f is onto

3) $g \subset f^{-1}(f(g))$ for any subset g of Xs

Cartesian product of fuzzy subsets: If μ_1 , μ_2 are two fuzzy subsets of X, $\mu 1 \times \mu 2$ is a fuzzy subset of X x X defined by $(\mu 1 \times \mu 2)(x_1, x_2) = \text{Min.}\{\mu_1(x_1), \mu_2(x_2)\}, (x_1, x_2)$ □ X x X.

UNIT-III

Assignment-Cum-Tutorial Questions

SECTION-A

Objective Questions

Given two fuzzy sets $A = \{\frac{0}{1} + \frac{0.25}{2} + \frac{0.3}{3} + \frac{0.4}{4}\}$ and $B = \{\frac{0}{1} + \frac{0.3}{2} + \frac{0.5}{3} + \frac{1}{4}\}$

1) A.B = ______________ [ ]

a) $\{\frac{0}{1} + \frac{0.75}{2} + \frac{0.15}{3} + \frac{0.4}{4}\}$ b) $\{\frac{0.4}{1} + \frac{0.75}{2} + \frac{0.4}{3} + \frac{0.15}{4}\}$ c) $\{\frac{0}{1} + \frac{0.75}{2} + \frac{0.4}{3} + \frac{0.15}{4}\}$ d)none

2) A+ B =___________ [ ]

a) $\{\frac{0}{1} + \frac{0.475}{2} + \frac{0.5}{3} + \frac{1}{4}\}$ b) $\{\frac{0}{1} + \frac{0.45}{2} + \frac{0.475}{3} + \frac{1}{4}\}$ c) $\{\frac{0}{1} + \frac{0.475}{2} + \frac{0.65}{3} + \frac{1}{4}\}$ d) none

3) A.0 =______________ [ ]

a) 0 b) 1 c) A d)none

4) A + 0= ___________ [ ]

a) A b) 0 c) 1 d)none

5) A + 1= _____________ [ ]

a)1 b)0 c) $\frac{0}{1} + \frac{0.25}{2} + \frac{0.3}{3} + \frac{0.4}{4}$ d)A

(common data to question no. 6 to 11)

Given $= \{\frac{0}{10} + \frac{0.2}{20} + \frac{.7}{30}\}$, $B = \{\frac{1}{10} + \frac{0.5}{20} + \frac{.3}{30}\}$ and $C = \{\frac{.4}{10} + \frac{0.8}{20} + \frac{1}{30}\}$ then

6. (A.B).C =_______________ [ ]

a) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ b) $\{\frac{0}{10} + \frac{0.2}{20} + \frac{.7}{30}\}$ c) $\{\frac{1}{10} + \frac{0.5}{20} + \frac{.3}{30}\}$ d)none

7. A.(B.C) = ____________ [ ]

a) $\{\frac{0}{10} + \frac{0.2}{20} + \frac{.7}{30}\}$ b) $\{\frac{1}{10} + \frac{0.5}{20} + \frac{.3}{30}\}$ c) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ d)none

8. $(A.B)^C$ = ________________ [ ]

a) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ b) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ c) $\{\frac{1}{10} + \frac{0.9}{20} + \frac{.79}{30}\}$ d)none

9. $A^C .B^C$ = _____________ [ ]

a) $\{\frac{1}{10} + \frac{0.8}{20} + \frac{.3}{30}\}$ b) $\{\frac{0}{10} + \frac{0.4}{20} + \frac{.21}{30}\}$ c) $\{\frac{0}{10} + \frac{0.5}{20} + \frac{.7}{30}\}$ d)none

10. $(A+B)^C$ = ______________ [ ]

a) $\{\frac{1}{10} + \frac{0.6}{20} + \frac{.79}{30}\}$ b) $\{\frac{0}{10} + \frac{0.4}{20} + \frac{.21}{30}\}$ c) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ d)none

11. $A^C + B^C$ =___________ [ ]

a) $\{\frac{0}{10} + \frac{0.08}{20} + \frac{.21}{30}\}$ b) $\{\frac{0}{10} + \frac{0.8}{20} + \frac{.2}{30}\}$ c) $\{\frac{1}{10} + \frac{0.9}{20} + \frac{.79}{30}\}$ d)none

12. If μ_1, μ_2 are two fuzzy subsets of X then $(\mu_1 . \mu_2)(x)$ =_______________

a) $\mu_1(x)\,\mu_2(x)$ b) $\mu_1(x) + \mu_2(x)$ c) $1+ \mu_1(x)\,\mu_2(x)$ d)none [ ]

13. $(\mu_1 + \mu_2)(x)$ = ____________________

a) $\mu_1(x) + \mu_2(x)$ b) $\mu_1(x) + \mu_2(x) - \mu_1(x)\,\mu_2(x)$ c) $\mu_1(x)\,\mu_2(x)$ d)none

14. $\mu . 0$ = ________ [ ]

a) 1 b) μ c)0 d)none

15. $(\mu_1 . \mu_2)^c$ = ____________ [ ]

a) $\mu_1^c . \mu_2^c$ b) $\mu_1^c . \mu_2$ c) $\mu_1 . \mu_2^c$ d) $\mu_1^c + \mu_2^c$

16. $(\mu_1 + \mu_2)^c$ = ____________ [ ]

a) $\mu_1^c . \mu_2^c$ b) $\mu_1 + \mu_2$ c) $\mu_1^c + \mu_2^c$ d) $\mu_1 . \mu_2$

17. Given A = $\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$ and B = $\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$ are fuzzy sets of X then (A.B) (x) =_________________ [ ]

a) $\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}$ b) $\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}$ c) $\frac{0.5}{2} + \frac{0.35}{3} + \frac{0.06}{4} + \frac{0.08}{5}$ d)None

18. Given A = $\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$ and B = $\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$ are fuzzy sets of X then (A+B) (x) =___________________ [ ]

a) $\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}$ b) $\frac{0.5}{2} + \frac{0.35}{3} + \frac{0.06}{4} + \frac{0.08}{5}$ c) $\frac{1}{2} + \frac{0.85}{3} + \frac{0.44}{4} + \frac{0.52}{5}$ d)None

SECTION-B

SUBJECTIVE QUESTIONS

1) List out the properties satisfied by fuzzy subsets under algebraic sum and product.
2) Show that $(\mu_1 + \mu_2) + \mu_3 = \mu_1 + (\mu_2 + \mu_3)$ i.e show that associativity holds under algebraic sum of fuzzy subsets.
3) Show that $(\mu_1 . \mu_2) . \mu_3 = \mu_1 . (\mu_2 . \mu_3)$ i.e show that associativity holds under algebraic product of fuzzy subsets.
4) Prove that commutativity holds under algebraic sum and product.
5) Does algebraic product distributes over algebraic sum of fuzzy sets justify your answer?
6) Given A = $\{\frac{.1}{10} + \frac{0.5}{20} + \frac{1}{30}\}$ and B= $\{\frac{.2}{10} + \frac{0.4}{20} + \frac{.8}{30}\}$ compute A.B
7) Given A = $\{\frac{.1}{10} + \frac{0.5}{20} + \frac{1}{30}\}$ and B= $\{\frac{.2}{10} + \frac{0.4}{20} + \frac{.8}{30}\}$ compute A+B.
8) Show that Demorgan's law holds under algebraic sum and algebraic product?
9) Show that A+1 = 1,where A is a fuzzy subset of X.
10) Show that A.1 = A, where A is A fuzzy subset of X.
11) Given A={(a,0.5),(b, 0.7),(c,0)} B= {(a,0.8),(b,0.2),(c,1)}, then Prove that
    i) A.B = B.A

    ii) A+B = B+A

Problems:

1)Let $X=\{x_1, x_2, x_3, x_4\}$ & $Y=\{y_1, y_2, y_3, y_4\}$ and $f:X\rightarrow Y$ defined by $f(x_1) = f(x_2) = y_1$, $f(x_3) = y_3$, $f(x_4) = y_4$. If $\mu=\{(y_1, 0.2), (y_2, 0.4), (y_3, 0.5), (y_4, 0.1)\}$ then find $f^{-1}(\mu)$.

2) If $f:X\rightarrow Y$ be a function. Prove that $f^{-1}(\mu^c) = (f^{-1}(\mu))^c$, for every fuzzy subset μ of Y.

3) If $f:X\rightarrow Y$ be a function. Prove that $(f(\mu))^c = f(\mu^c)$ for every fuzzy subset μ of X if f is onto

4) Given A = $\{\frac{0.5}{1} + \frac{0.6}{2} + \frac{0.8}{3} + \frac{1}{4} + \frac{1}{5}\}$ B = $\{\frac{0.2}{1} + \frac{0.4}{2} + \frac{0.7}{3} + \frac{0.9}{4} + \frac{1}{5}\}$ are two fuzzy sets of X show that i)$(A.B)^C = A^C + B^C$ and ii) $(A + B)^C = A^C . B^C$

5) show that $(A.B)^C = A^C + B^C$ and ii) $(A + B)^C = A^C . B^C$ by using definition of algebraic product and sum.

6) Given A = $\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$, B = $\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$ and C = $\{\frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{1}{5}\}$ show that A.(B.C) = (A.B).C and represent it graphically.

7) Given A = $\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$, B = $\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$ and C = $\{\frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{1}{5}\}$ show that A+ (B + C) = (A+ B) + C and represent it graphically.

8) Given A = $\{\frac{1}{2} + \frac{0.5}{3} + \frac{0.3}{4} + \frac{0.2}{5}\}$, B = $\{\frac{0.5}{2} + \frac{0.7}{3} + \frac{0.2}{4} + \frac{0.4}{5}\}$ and C = $\{\frac{0.1}{2} + \frac{0.4}{3} + \frac{0.5}{4} + \frac{1}{5}\}$

i) $(A.B)^C = A^C + B^C$ ii) $(A+B)^C = A^C . B^C$

UNIT-IV

Course Objectives: The objectives of this course are to

- Provide an emphasis on the differences between crisp relation and fuzzy relation.
- Become familiar with operations on fuzzy relations and properties of fuzzy relations.
- Provide a brief introduction on logic and logical connectives .

Syllabus: fuzzy relations, algebra of fuzzy relations, properties of fuzzy relations, logic, connectives.

Course Outcomes: At the end of the course students should be able to

- Distinguish between crisp relation and fuzzy relation.
- Draw parallelism between properties of crisp relations and properties of fuzzy relations.
- Know the use of logic and logical connectives.

Learning material

Fuzzy relation: A fuzzy relation is a mapping from the Cartesian space X x Y to the interval [0 , 1] where the strength of the mapping is expressed by the membership function of the relation $\mu_R(x, y)$.

$\mu_R : X \times Y \to [0,1] R = \{((x, y), \mu_R(x, y)) / \mu_R(x, y) \geq 0 x \in X, y \in Y\}$

OR

Fuzzy relation is characterized by a function $\mu_R : X_1 \times ... \times X_m \to [0,1]$ where X_i are the universes of discourse and $X_1 \times ... \times X_m$ is the product space. If we have two finite universes, the fuzzy relation can be presented as a matrix (*fuzzy matrix*) whose elements are the intensities of the relation and $\boldsymbol{R}$ has the membership function $\mu_R(x, y) where\ x \in X_{1,} y \in X_2$.

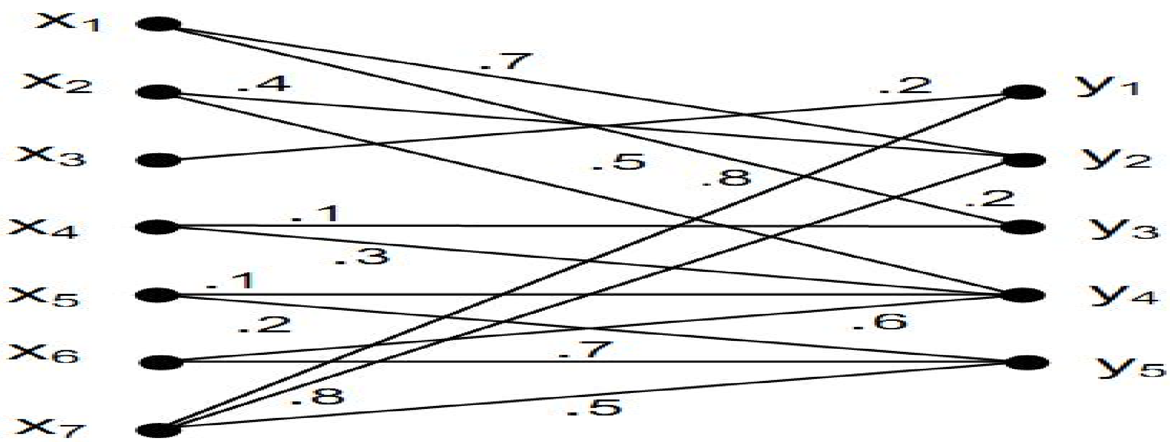
OR

fuzzy relation is a fuzzy set defined on tuples $(x_1, \ldots, x_n)$ that may have varying degrees of membership within the relation.

The membership grade indicates strength of the present relation between elements of the tuple.

Example: 1)A fuzzy relation "FRIEND" describes the degree of friend ship between two persons (in contrast to either being friend or not being friend in classical relation)

2)



3)

Let R be a fuzzy relation between two sets $X = \{\text{New York City}, \text{Paris}\}$ and $Y = \{\text{Beijing}, \text{New York City}, \text{London}\}$.

R shall represent relational concept "very far".

This relation can be written in a list notation as

$$R(X, Y) = 1/\text{NYC, Beijing} + 0/\text{NYC, NYC} + 0.6/\text{NYC, London} + 0.9/\text{Paris, Beijing} + 0.7/\text{Paris, NYC} + 0.3/\text{Paris, London}.$$

It can be also represented as two-dimensional membership array:

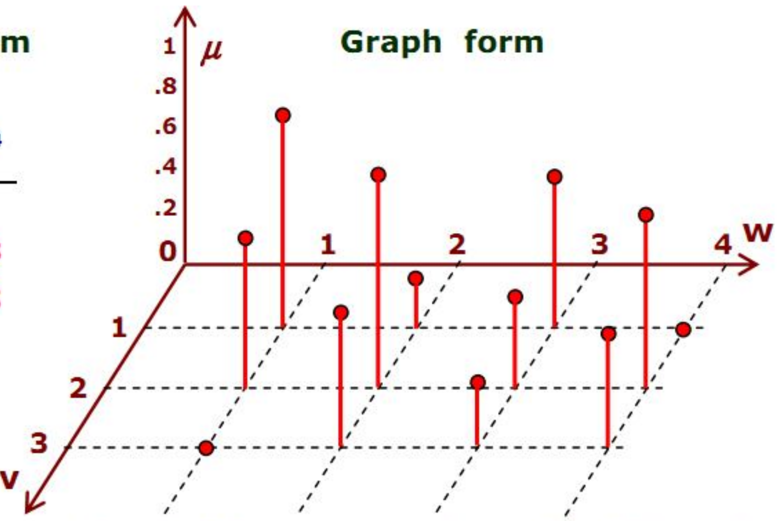
	NYC	Paris
Beijing	1	0.9
NYC	0	0.7
London	0.6	0.3

4)

Membership matrix form

$R \triangleq$

v \ w	w_1	w_2	w_3	w_4
v_1	1	0.2	0.7	0
v_2	0.7	1	0.4	0.8
v_3	0	0.6	0.3	0.5



Vertical lines represent membership grades

Domain of a fuzzy relation:

Given a fuzzy relation $R(X, Y)$.

Its **domain** dom R is the fuzzy set on X whose membership function is defined for each $x \in X$ as

$$\text{dom}\, R(x) = \max_{y \in Y} R(x, y),$$

i.e. each element of X belongs to the domain of R to a degree equal to the strength of its strongest relation to any $y \in Y$.

Range of a fuzzy relation:

The **range** ran of $R(X, Y)$ is a fuzzy relation on Y whose membership function is defined for each $y \in Y$ as

$$\text{ran}\, R(y) = \max_{x \in X} R(x, y),$$

i.e. the strength of the strongest relation which each $y \in Y$ has to an $x \in X$ equals to the degree of membership of y in the range of R.

Example:

$$R = \begin{array}{c|ccccc} & y_1 & y_2 & y_3 & y_4 & y_5 \\ \hline x_1 & .9 & 1 & 0 & 0 & 0 \\ x_2 & 0 & .4 & 0 & 0 & 0 \\ x_3 & 0 & .5 & 1 & .2 & 0 \\ x_4 & 0 & 0 & 0 & 1 & .4 \\ x_5 & 0 & 0 & 0 & 0 & .5 \\ x_6 & 0 & 0 & 0 & 0 & .2 \end{array}$$

$\mu_{dom(R)}(x_1) = 1.0$
$\mu_{dom(R)}(x_2) = 0.4$
$\mu_{dom(R)}(x_3) = 1.0$
$\mu_{dom(R)}(x_4) = 1.0$
$\mu_{dom(R)}(x_5) = 0.5$
$\mu_{dom(R)}(x_6) = 0.2$

Height of a fuzzy relation: The height of a fuzzy relation R is denoted by h(R) and is defined as the largest membership grade attained by any pair (x,y) of R.

OR

The **height** h of $R(X, Y)$ is a number defined by

$$h(R) = \max_{y \in Y} \max_{x \in X} R(x, y).$$

$h(R)$ is the largest membership grade obtained by any pair $(x, y) \in R$

NOTE: If h(R) =1 it is called normal otherwise subnormal.

Inverse of a fuzzy relation: The inverse of a fuzzy relation R(x,y) is given by

$\mu_{R^{-1}}(y, x) = \mu_R(x, y)$

Support of a fuzzy relation: consider a fuzzy relation R .The support of R is denoted by S(R) and is defined as $S(R) = \{(x, y) / \mu_R(x, y) > 0\}$

Closet: Let R be a fuzzy relation .ordinary relation $\underline{R}$ Closet to R is given by

$$\mu_{\underline{R}}(x,y)=\begin{cases}0\; if\;\; \mu_R(x,y)<0.5\\ 1 if\;\; \mu_R(x,y)>0.5\\ 0\, or 1\, if \mu_R(x,y)=0.5\end{cases}$$

Note: Usually we take $\mu_{\underline{R}}(x,y)=0$ if $\mu_R(x,y)$=0.5

$Level-\alpha$:Consider a fuzzy relation R. The $level-\alpha\ of\ R_\alpha\ is G_\alpha\ and\ is\ defined\ as\ G_\alpha=\{(x,y)/$ $\mu_R(x,y)\geq\alpha\}$ where $\alpha\epsilon$[0,1]

Example:

0.9	0.4	0.0
0.2	1.0	0.4
0.0	0.7	1.0
0.4	0.2	0.0

$M_{R\,0.9}=$

1	0	0
0	1	0
0	0	1
0	0	0

$M_{R\,1.0}=$

0	0	0
0	1	0
0	0	1
0	0	0

Algebra of fuzzy relations:

Let R and S be any two fuzzy relations from X to Y.

$$Union: \mu_{R\cup S}(x,y) = \max\{\mu_R(x,y), \mu_S(x,y)\}$$

$$Intersection: \mu_{R\cap S}(x,y) = \min\{\mu_R(x,y), \mu_S(x,y)\}$$

$$Algebraic product: \mu_{R\bullet S}(x,y) = \mu_R(x,y)\mu_S(x,y)$$

$$Algebraic sum: \mu_{R+S}(x,y) = \mu_R(x,y) + \mu_S(x,y) - \mu_R(x,y)\mu_S(x,y)$$

$$complement: \mu_{R^C}(x,y) = 1 - \mu_R(x,y)$$

Composition of fuzzy relations: Consider two fuzzy relations R(X x Y) and

S(Y x Z) then a relation T(X x Z) can be expressed as T = R o S

$$\mu_T(x,z) = \max - \min\{\mu_R(x,y), \mu_S(y,z)\}$$

Note: 1) The max-min composition can be interpreted as indicating the strength of the existence of relation between the elements of X and Z.

2) Calculation of ROS is almost similar to matrix multiplication

3) If algebraic product is adopted , then max-product composition is adopted

$$T = ROS = \mu_T(x,z) = \max\{\mu_R(x,y).\mu_S(y,z)\}$$

Example:1)

$$P \circ Q = R$$

$$\begin{bmatrix} .3 & .5 & .8 \\ 0 & .7 & 1 \\ .4 & .6 & .5 \end{bmatrix} \circ \begin{bmatrix} .9 & .5 & .7 & .7 \\ .3 & .2 & 0 & .9 \\ 1 & 0 & .5 & .5 \end{bmatrix} = \begin{bmatrix} .8 & .3 & .5 & .5 \\ 1 & .2 & 5 & .7 \\ .5 & .4 & .5 & .5 \end{bmatrix}$$

For instance:

$$\begin{aligned} r_{11} &= \max\{\min(p_{11}, q_{11}), \min(p_{12}, q_{21}), \min(p_{13}, q_{31})\} \\ &= \max\{\min(.3, .9), \min(.5, .3), \min(.8, 1)\} \\ &= .8 \\ r_{32} &= \max\{\min(p_{31}, q_{12}), \min(p_{32}, q_{22}), \min(p_{33}, q_{32})\} \\ &= \max\{\min(.4, .5), \min(.6, .2), \min(.5, 0)\} \\ &= .4 \end{aligned}$$

2)

Consider the following fuzzy relations for airplanes:

- relation A between maximal speed and maximal height,
- relation B between maximal height and the type.

A	h_1	h_2	h_3
s_1	1	.2	0
s_2	.1	1	0
s_3	0	1	1
s_4	0	.3	1

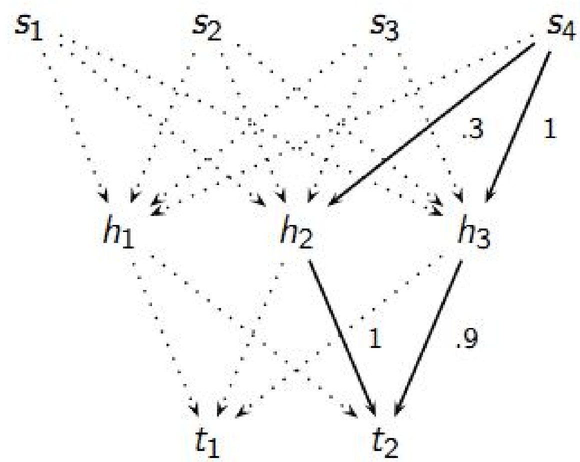
B	t_1	t_2
h_1	1	0
h_2	.9	1
h_3	0	.9

matrix multiplication scheme

			1	0
A	$\circ$	***B***	.9	1
			0	.9
1	.2	0	1	.2
.1	1	0	.9	1
0	1	1	.9	1
0	.3	1	.3	.9

$A \circ B$ speed-type relation

flow scheme



$$(A \circ B)(s_4, t_2) = \max\{\min\{.3,\ 1\},\ \min\{.1,\ .9\}\}$$
$$= .9$$

3)

Speed of bowling = {fast bowling, medium bowling, spin bowling} and

Y= condition on pitches= {good wicket, fair wicket, sporting wicket, green wicket, crumbling wicket, rough wicket}

Let R denotes the relationship between speed of bowling and condition on pitch and Q denotes the relationship between conditions on pitches and runs on the board.

$$R = \begin{array}{c} fast \\ medium \\ spin \end{array} \overset{\begin{array}{cccccc} gd.w & f.w & s.w & gr.w & c.w & r.w \end{array}}{\begin{bmatrix} 0.6 & 0.5 & 0.4 & 0.1 & 0.9 & 0.5 \\ 0.8 & 0.6 & 0.9 & 0.2 & 0.1 & 0.6 \\ 0.7 & 0.8 & 0.6 & 0.7 & 0.1 & 0.2 \end{bmatrix}}$$

and

$$Q = \begin{array}{c} gd.w \\ f.w \\ s.w \\ gr.w \\ c.w \\ r.w \end{array} \overset{\begin{array}{ccc} low.r & ave.r & hig.r \end{array}}{\begin{bmatrix} 0.4 & 0.8 & 0.7 \\ 0.3 & 0.8 & 0.8 \\ 0.2 & 0.7 & 0.8 \\ 0.8 & 0.6 & 0.4 \\ 0.7 & 0.5 & 0.4 \\ 0.9 & 0.4 & 0.2 \end{bmatrix}}$$

$R \circ Q$ = Relationship between speed of the bowling and runs on the board

We calculate $R \circ Q$ by using max-min composition rule

$$\max\{\min(0.6,0.4),\min(0.5,0.3),\min(0.4,0.2),\min(0.1,0.8),\min(0.9,0.7),\min(0.5,0.9)\}$$

$$= \max\{0.4,0.3,0.2,0.1,0.7,0.5\}$$

$$= 0.7$$

Similarly we can calculate the other entries

The relational matrix for max-min composition in fuzzy relational is thus

$$R \circ Q = \begin{array}{c} fast \\ medium \\ spin \end{array} \overset{\begin{array}{ccc} low.r & ave.r & hig.r \end{array}}{\begin{bmatrix} 0.7 & 0.6 & 0.6 \\ 0.6 & 0.8 & 0.8 \\ 0.7 & 0.8 & 0.8 \end{bmatrix}}$$

Max Product composition

Now by using max product composition we find the relationship between speed of the bowling and runs on the board

$R \circ Q =$ Relationship between speed of the bowling and runs on the board

We calculate $R \circ Q$ by using max product composition rule

$$\max(0.6.0.4, 0.5.0.3, 0.4.0.2, 0.1.0.8, 0.9.0.7, 0.5.0.9)$$

$$= \max(0.24, 0.15, 0.08, 0.08, 0.63, 0.45)$$

$$= 0.63$$

Similarly we calculate the other entries and the relational matrix for max product

$$R \circ Q = \begin{array}{c} \\ fast \\ medium \\ spin \end{array} \begin{array}{c} \begin{array}{ccc} low.r & ave.r & hig.r \end{array} \\ \begin{bmatrix} 0.63 & 0.48 & 0.4 \\ 0.54 & 0.64 & 0.64 \\ 0.56 & 0.64 & 0.64 \end{bmatrix} \end{array}$$

4)

Let $R_1(x,y)$ *and* $R_2(x,y)$ be defined by the following relational matrix

$$R_1 = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccccc} y_1 & y_2 & y_3 & y_4 & y_5 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0 & 1 & 0.7 \\ 0.3 & 0.5 & 0 & 0.2 & 1 \\ 0.8 & 0 & 1 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

$$R_2 = \begin{array}{c} \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array}\begin{array}{c} \begin{array}{cccc} z_1 & z_2 & z_3 & z_4 \end{array} \\ \begin{bmatrix} 0.9 & 0 & 0.3 & 0.4 \\ 0.2 & 1 & 0.8 & 0 \\ 0.8 & 0 & 0.7 & 1 \\ 0.4 & 0.2 & 0.3 & 0 \\ 0 & 1 & 0 & 0.8 \end{bmatrix} \end{array}$$

we shall first compute the max-min composition $R_1 \circ R_2(x,z)$

$$\mu_{R_1 \circ R_2}(x_1, z_1) = \max(\min(0.1, 0.9), \min(0.2, 0.2), \min(0, 0.8), \min(1, 0.4), \min(0.7, 0))$$

$$= \max(0.1, 0.2, 0, 0.4, 0) = 0.4$$

Similarly we can determine the grades of membership for all pairs

$(x_i, z_i), i = 1, 2, 3, j = 1, \ldots\ldots 4$

$$R_1 o R_2 = \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \begin{bmatrix} 0.4 & 0.7 & 0.3 & 0.7 \\ 0.3 & 1 & 0.5 & 0.8 \\ 0.8 & 0.3 & 0.7 & 1 \end{bmatrix}$$

for the max product composition, we calculate

$$\mu_{R_1}(x_1, y_1).\mu_{R_2}(y_1, z_1) = 0.1.0.9 = 0.09$$

$$\mu_{R_1}(x_1, y_2).\mu_{R_2}(y_2, z_1) = 0.2.0.2 = 0.04$$

$$\mu_{R_1}(x_1, y_3).\mu_{R_2}(y_3, z_1) = 0.0.8 = 0$$

$$\mu_{R_1}(x_1, y_4).\mu_{R_2}(y_4, z_1) = 1.0.4 = 0.4$$

$$\mu_{R_1}(x_1, y_5).\mu_{R_2}(y_5, z_1) = 0.7.0 = 0$$

hence

$$\mu_{R_1 o R_2}(x_{1,} z_1) = \max\{0.09, 0.04, 0, 0.4, 0\} = 0.4$$

In the similar way after performing the remaining computation, we obtain

$$R_1 o R_2 = \begin{matrix} \\ x_1 \\ x_2 \\ x_3 \end{matrix} \begin{matrix} z_1 & z_2 & z_3 & z_4 \\ \left[\begin{matrix} 0.4 \\ 0.27 \\ 0.8 \end{matrix}\right. & \begin{matrix} 0.7 \\ 1 \\ 0.3 \end{matrix} & \begin{matrix} 0.3 \\ 0.4 \\ 0.7 \end{matrix} & \left.\begin{matrix} 0.56 \\ 0.8 \\ 1 \end{matrix}\right] \end{matrix}$$

Properties of fuzzy relation: Let R be fuzzy relation defined on X.

Reflexive: R is called reflexive iff $\mu_R(x, x) = 1 \ \forall x \in X$.

Example:

Let $X = \{1, 2, 3, 4\}$

$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0.9 & 0.6 & 0.2 \\ 0.9 & 1 & 0.7 & 0.3 \\ 0.6 & 0.7 & 1 & 0.9 \\ 0.2 & 0.3 & 0.9 & 1 \end{bmatrix} \end{array}$$

is reflexive relation

Symmetric: R is called symmetric iff $\mu_R(x, y) = \mu_R(y, x) \forall x, y \in X$

Example:

Let $X = \{x_1, x_2, x_3\}$

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0.8 & 0.1 & 0.7 \\ 0.1 & 1 & 0.6 \\ 0.7 & 0.6 & 0.5 \end{bmatrix} \end{array}$$ is a symmetric relation.

Antisymmetric: R is called antisymmetri iff $\mu_{\underline{R}}(x, y) > 0, \mu_{\underline{R}}(y, x) > 0 \Rightarrow x = y$
$\forall x, y \in X$.

Example:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0.7 \\ 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix} \end{array} \text{ is antisymmetric relation.}$$

Asymmetric: R is called asymmetric iff $\exists\, x, y,\ \mu_{\underline{R}}(x, y) \neq \mu_{\underline{R}}(y, x)\ \ \forall\, x, y \in X$

Transitive: R is called transitive iff
$\mu_{\underline{R}}(x, y) = \alpha \text{ and } \ \mu_{\underline{R}}(y, z) = \beta \rightarrow \mu_{\underline{R}}(x, z) = \lambda \text{ where } \lambda \geq \min\{\alpha, \beta\}\ \ \forall\, x, y, z \in X$

Example: x is a distance relative of y.

Note: Transitivity is also called as max –min transitivity. The strength of the link between 2 elements must be greater than or equal to the strength of the any indirect chain involving other elements.

Fuzzy equivalence relation: A fuzzy binary relation is called a fuzzy equivalence relation if it is reflexive, symmetric and transitive.

Example:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{bmatrix} 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ 0.2 & 1 & 0.2 & 0.2 & 0.8 & 0.2 \\ 1 & 0.2 & 1 & 0.6 & 0.2 & 0.6 \\ 0.6 & 0.2 & 0.6 & 1 & 0.2 & 0.8 \\ 0.2 & 0.8 & 0.2 & 0.2 & 1 & 0.2 \\ 0.6 & 0.2 & 0.6 & 0.8 & 0.2 & 1 \end{bmatrix} \end{array} \text{ is a similarity relation.}$$

Fuzzy compatibility relation: A fuzzy binary relation is called a fuzzy compatibility relation if it is reflexive, and symmetric.

Example:

$$R = \begin{bmatrix} 1 & 0.1 & 0.8 & 0.2 & 0.3 \\ 0.1 & 1 & 0 & 0.3 & 1 \\ 0.8 & 0 & 1 & 0.7 & 0 \\ 0.2 & 0.3 & 0.7 & 1 & 0.6 \\ 0.3 & 1 & 0 & 0.6 & 1 \end{bmatrix} \quad \text{is weak similarity relation}$$

Fuzzy preorder relation: A fuzzy binary relation is called a fuzzy preorder relation if it is reflexive and transitive.

Example:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 1 & 0.7 & 0.8 & 0.5 & 0.5 \\ 0 & 1 & 0.3 & 0 & 0.2 \\ 0 & 0.7 & 1 & 0 & 0.2 \\ 0.6 & 1 & 0.9 & 1 & 0.6 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

is a fuzzy preorder relation

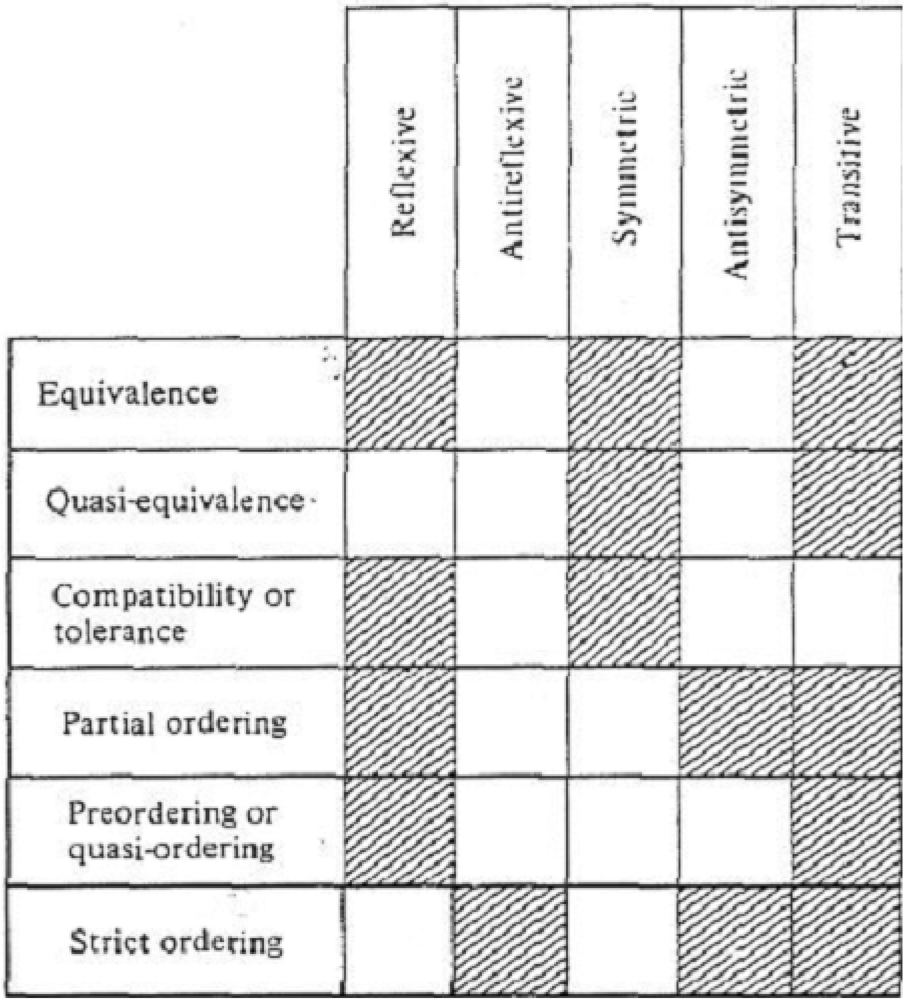
Fuzzy partial order relation: A fuzzy binary relation is called a fuzzy partial order relation if it is reflexive, anti-symmetric and transitive.

Example:

$$\mathbf{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & .7 & 0 & 1 & .7 \\ 0 & 1 & 0 & .9 & 0 \\ .5 & .7 & 1 & 1 & .8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & .1 & 0 & .9 & 1 \end{bmatrix} \end{array}$$

is a fuzzy partial ordering relation

Note:



Fuzzy poset : A set X along with a fuzzy partial ordering defined on it is called fuzzy poset.

Dominating class: The dominating class of x is denoted by $R_{\geq[X]}$ contains the members of X to the degree to which they dominate x.

The class dominated by x is denoted by $R_{\leq[X]}$ contains the elements of X to the degree to which they are dominated by x.

Un dominated: An element $x \in X$ is un dominated iff $\mu_{\underline{R}}(x, y) = 0 \forall y \in X \; and x \neq y$

Fuzzy upper bound: Let X be a fuzzy poset with a fuzzy partial order R and A a crisp subset of X .The fuzzy upper bound for A is the fuzzy set denoted by U(R,A) and is defined by U(R,A) = $\bigcap_{x \in A} R_{\geq[X]}$ where $\cap$ denotes fuzzy intersection. If a least upper bound of the set A exists, it is the unique element x in U(R, A) $\ni \mu_{U(R,A)}(x)$ >0 and $\mu_R(x, y)$ > 0 for each y in the support of U(R,A).Similarly fuzzy lower bound.

Example:

$$\mathbf{R} = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{bmatrix} 1 & .7 & 0 & 1 & .7 \\ 0 & 1 & 0 & .9 & 0 \\ .5 & .7 & 1 & 1 & .8 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & .1 & 0 & .9 & 1 \end{bmatrix} \end{array}$$

The dominating class for each element is given by the row of the matrix corresponding to that element. The columns of the matrix give the dominated class for each element. Under this ordering, the element d is undominated, and the element c is undominating. For the subset $A = \{a, b\}$, the upper bound is the fuzzy set produced by the intersection of the dominating classes for a and b. Employing the min operator for fuzzy intersection, we obtain

$$U(R, \{a, b\}) = .7/b + .9/d.$$

Fuzzy linear order: A fuzzy linear order R on as et X is afuzzy partial order $\ni \forall x, y \, if x \neq y \, then \; either \; \mu_R(x, y) > 0 \, or \; \mu_R(y, x) > 0.$

Fuzzy lattice: A fuzzy poset X is called a fuzzy lattice if every two element non fuzzy set in X has fuzzy upper bound and fuzzy lower bound.

Logic: Logic is the science which deals with methods of reasoning.In order to study logic, we first develop formal language called object language.A logic is based on two truth values 0 and 1 is sometimes in adequate for describing human reasoning.

Fuzzy logic uses whole interval between 0 and 1 to describe human reasoning.It refers to a logic of approximation.It was created by a Dr.

Lotfi Zadeh in 1960' for the purose of modeling the uncertainity in herent in natural language.It is a multivalued logic.

Statement:

A declarative sentence which is either true or false but not both is called a statement or proposition.

- Statements are generally denoted by either upper case or lower case letters.

Examples:

1. Bombay is the capital of Canada (Statement)
2. Canada is a country (Statement)
3. 10 + 100 = 110 (Statement)
4. 3 + 3 = 4 (Statement)
5. x + 5 =8 (not a statement)
6. close the door (not a statement)
7. what is your name? (not a statement)

Atomic statement:

A statement that cannot be broken down into more than one simpular statement is called atomic or primary or primitive statement.

Compound statement:

A statement that can be broken down into simpular statements is called compound or molecular statement.

Propositional calculus:

The area of logic that deals with propositions is called propositional calculus or propositional logic.

Truth value:

- Truth value for true statement is T
- Truth value for false statement is F

Statement	**Truth value**
Bombay is the capital of Canada	F
Canada is a country	T
10 + 100 = 110	T
3 + 3 = 4	F
x + 5 =8	Not a statement We can't give truth value

Connectives:

The words or expressions which are used to construct compound statements from simpular statements are known as sentential connectives.

- And, or, if then, iff, not, so, because are sentential connectives.
- $+, -, \times, \div, \cup, \cap, \leq, \geq, <, >$ are mathematical connectives.

Different types of compound statements:

Type	Connective	Symbol	Notation	Read as
Conjunction	And	$\wedge$	$P \wedge Q$	P and Q
Disjunction	Or	$\vee$	$P \vee Q$	P or Q
Conditional	If then	$\longrightarrow$	$P \longrightarrow Q$	P implies Q i.e If P then Q
Bi-conditional	If and only if	$\leftrightarrow$	$P \leftrightarrow Q$	P double implies Q i.e. P if and only if Q
Negation	Not or No	$\backsim$ or □	$\backsim P$	Negation P (or) Not P

Truth Table:

The table showing the truth values of a statement formula for each possible combination of the truth values of the compound statements is called the truth table of the formula.

Note: In general if there are n distinct components in a statement formula we need to consider 2^n possible combinations of truth values in order to construct the truth table.

Truth table rules:

P	Q	$P \wedge Q$	$P \vee Q$	$P \longrightarrow Q$	$P \leftrightarrow Q$	$\backsim P$
T	T	T	T	T	T	F
T	F	F	T	F	F	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

Problems:

Q. Using the statements R: Mark is rich H: Mark is happy

Denote the following statements in symbolic form.

a) Mark is poor but happy. **Ans:** $\backsim R \wedge H$
b) Mark is rich or unhappy. **Ans:** $R \vee \backsim H$
c) Mark is neither rich nor happy. **Ans:** $\backsim R \wedge \backsim H$
d) Mark is poor or he is both rich and unhappy. **Ans:** $\backsim R \vee (R \wedge \backsim H)$

Q. Represent the following statement in symbolic form.

" If either John takes Computer science or Merin takes Mathematics then Nishanth will take Biology."

Ans: Let us denote the statements as follows.

P: John takes Computer science

Q: Merin takes Mathematics

R: Nishanth takes Biology

Then given statement can be written as $(P \vee Q) \longrightarrow R$

Q. How can the following statement be translated into a logical expression.

" You can access the internet from campus only if you are a Computer science major student or you are not a freshman."
(For student)

Q. Construct the truth tables for the following statement formula.

1. $\backsim(\backsim P \vee \backsim Q)$

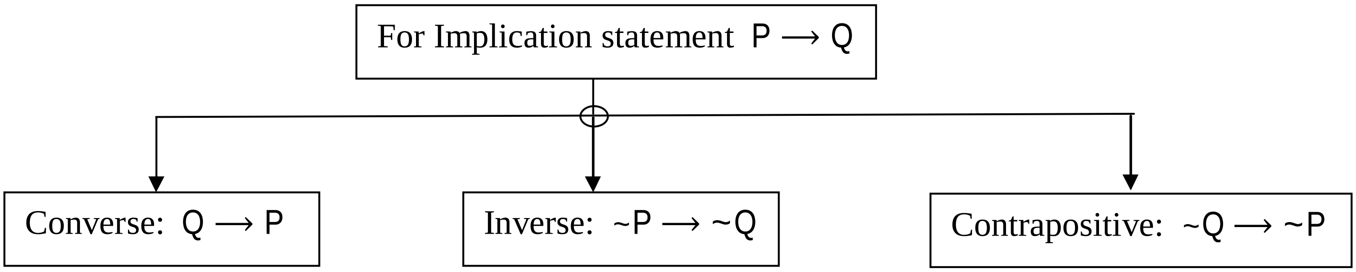
P	Q	$\backsim P$	$\backsim Q$	$\backsim P \vee \backsim Q$	$\backsim(\backsim P \vee \backsim Q)$
T	T	F	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

Q. $(\square P \wedge (\square Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$

P	Q	R	$\square P$	$\square Q$	$\square Q \wedge R$	$\square P \wedge (\square Q \wedge R)$ A	$Q \wedge R$ B	$P \wedge R$ B	$A \vee B$	$A \vee B \vee C$
T	T	T	F	F	F	F	T	T	T	T
T	T	F	F	F	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T	F	T
T	F	F	F	T	F	F	F	F	F	F
F	T	T	T	F	F	F	T	F	T	T
F	T	F	T	F	F	F	F	F	F	F
F	F	T	T	T	T	T	F	F	T	T
F	F	F	T	T	F	F	F	F	F	F

Q. $[Q \wedge (P \longrightarrow Q)] \rightarrow P$ (For student)

Q. $\sim(P \wedge Q) \leftrightarrow \sim P \vee \sim Q$ (For student)



Note: Converse of inverse of an implication is a contrapositive.

Inverse of connverse of an implication is a contrapositive.

Conditional	$(P \longrightarrow Q)$	If I am sleeping, then I am breathing
Converse	$(Q \longrightarrow P)$	If I am breathing, then I am sleeping
Inverse	$(\sim P \longrightarrow \sim Q)$	If I am not sleeping, then I am not breathing
Contrapositive	$(\sim Q \longrightarrow \sim P)$	If I am not breathing, then I am not sleeping

Q.What are the converse, inverse and contrapositive of the implication "If I get good.

rank in EAMCET then I will choose CSE. "

Ans: Let us take the statements as follows.

P: I get good rank in EAMCET

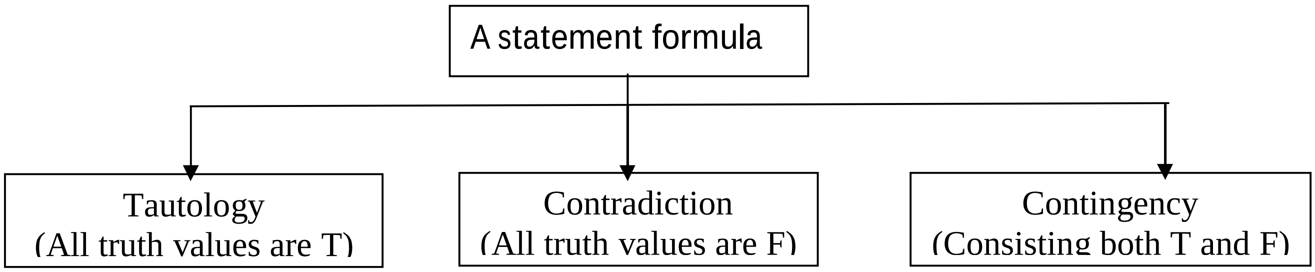
Q: I will choose CSE

Converse: If I choose CSE, then I got good rank in EAMCET.

Inverse : If I not get good rank in EAMCET , then I will not choose CSE.

Contrapositive: If I will not choose CSE, then I did not get good rank in EAMCET.

Q. What are the inverse, converse, and contrapositive of the implication "If today is a holiday, then I will go for a movie "
(For student)



Tautology:

A statement formula that is always true, irrespective of the truth values of the propositions that occur in it, is called a tautology. This is also called as universally valid formula or a logical truth.

Contradiction:

A statement formula that is always false, irrespective of the truth values of the propositions that occur in it, is called contradiction.

Contingency:

A proposition that is neither a tautology nor a contradiction is called a contingency.

Note: 1. The negation of contradiction is tautology.

2. The conjunction of two tautologies is also a tautology.

Q. Indentify that $[(p \longrightarrow (q \longrightarrow r)) \longrightarrow ((p \longrightarrow q) \longrightarrow (p \longrightarrow r))]$ is a tautology.

p	**q**	**R**	$p \longrightarrow q$ **A**	$q \longrightarrow r$ **B**	$p \longrightarrow r$ **C**	$p \longrightarrow B$ **D**	$A \longrightarrow C$ **E**	$D \longrightarrow E$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

From above truth table,

Hence the given statement is Tautology.

Q. Show that $((\neg q \wedge p) \wedge q)$ is a contradiction.

P	**q**	$\neg q$	$\neg q \wedge p$	$((\neg q \wedge p) \wedge q)$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

From above truth table,

Hence the given statement is Contradiction.

Equivalence of formalas:

The two propositions A and B are said to be logically equivalent if $A \leftrightarrow B$ is a tautology.

And written as $A \Leftrightarrow B$ and read as A is equivalent to B.

Q. show that $(P \longrightarrow Q) \leftrightarrow (\sim P \vee Q)$

P	**Q**	$\sim \boldsymbol{P}$	$\boldsymbol{P \longrightarrow Q}$	$\sim \boldsymbol{P \vee Q}$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From the above truth table,

Hence $(P \longrightarrow Q) \leftrightarrow (\sim P \vee Q)$

Q. Show that $((P \wedge \square P) \vee Q) \Leftrightarrow Q$

P	**Q**	$\neg \boldsymbol{P}$	$\boldsymbol{P \wedge \square P}$	$\boldsymbol{(P \wedge \square P) \vee Q}$
T	T	F	F	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	F

From the above truth table,

Hence $((P \wedge \square P) \vee Q) \Leftrightarrow Q$

Equivalent formulae:

The logical equivalences below are important equivalences that should be memorized.

Idempotent Laws: $p \vee p \Leftrightarrow p$

$p \wedge p \Leftrightarrow p$

Commutative Laws: $p \vee q \Leftrightarrow q \vee p$

$p \wedge q \Leftrightarrow q \wedge p$

Associative Laws: $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Distributive Laws: $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Identity Laws: $p \wedge T \Leftrightarrow p$

$p \vee F \Leftrightarrow p$

Domination Laws: $p \vee T \Leftrightarrow T$

$p \wedge F \Leftrightarrow F$

De Morgan's Laws: $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Absorption Laws: $p \wedge (p \vee q) \Leftrightarrow p$

$p \vee (p \wedge q) \Leftrightarrow p$

Negation Laws: $p \vee \neg p \Leftrightarrow T$

$p \wedge \neg p \Leftrightarrow F$

Double Negation Law: $\neg(\neg p) \Leftrightarrow p$

Dual Formulas:

Two formulas A and A^* are said to be duals of each other if either one can be obtained from the other by replacing $\wedge$ by $\vee$,$\vee$ by $\wedge$,T by F,F by T.

Duality Law:

If any two formulas are equivalent then their duals are also equivalent to each other.

i.e., $A \Leftrightarrow B$ $then$ $A^* \Leftrightarrow B^*$

Q. Write the duals of $(i).(P \wedge Q) \vee T$

$(ii).\ \neg(P \vee Q) \wedge (P \vee (\neg(Q \wedge \neg S))).$

Ans: Dual of given formulas are

$(i).\ (P \vee Q) \wedge F$

$(ii).\ \neg(P \wedge Q) \vee (P \wedge (\neg(Q \vee \neg S))).$

Functionally complete:A set of connectives is called functionally complete if every formula can be expressed in terms of an equivalent formula containing the connectives from this set

UNIT-IV
Assignment-Cum-Tutorial Questions
SECTION-A

Objective Questions

1. State Duality law.
2. Define level-α of a fuzzy relation.
3. Is Pv7P a tautology?
4. If h(R) = 1, then the fuzzy relation R is called__________
5. If h(R) ≠ 1then the fuzzy relation R is called__________
6. $\mu_{R\cup S}(x,y) =$ _________________
7. $\mu_{R\cap S}(x,y) =$ __________________
8. $\mu_{R\bullet S}(x,y) ==$_________________
9. $\mu_{R+S}(x,y) ==$________________
10. $\mu_{R^C}(x,y) =$ _______________
11. If $\mu_R(x,x)$ = 1 $\forall x \in X$, where R is a fuzzy relation on X then R is called_________
12. Transitivity is also called as ________________
13. Let R be a fuzzy relation on X, if $\mu_R(x,y) = \mu_R(y,x) \forall x,y \in X$ *then R is called*_________
14. If a fuzzy relation R is reflexive, anti symmetric and transitive then R is called________
15. If a fuzzy relation R is reflexive and transitive then R is called________
16. If a fuzzy relation R is reflexive and symmetric then R is called________
17. Let R(XxY) and S (YxZ) be two fuzzy relations then $\mu_{I=R0S}(x,z) =$ ________

2) If p and q are two statements then the converse of $\neg q \Rightarrow \neg p$

3) The inverse of (p $\wedge$ q) $\Rightarrow$ ($\neg q \wedge \neg p$) is

4) The negation of the statement ‘ there are 7 days in a week ‘ is

5) What is the truth value of the statement ‘If charjminar is in Hyderabad then 5*3=8

6) What is the equivalent formulae of $\neg\forall x(p(x))$?

7) Which of the following is a tautology ?

a) $\neg p \Rightarrow (p \wedge q)$ b) $(((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (p)) \Rightarrow r$

c) $p \Rightarrow p \vee q$ d) $p \wedge q$

8) If the truth value of q is T then the truth value of $(q \vee r) \wedge q$?

9) The truth value of 2+6=9 if and only if 9+6=10 is ?

10) Which of the following is a contingency?

a) $(p \wedge q) \Rightarrow (p \vee q)$ b) $p \vee q \Rightarrow (p \wedge q)$

c) $p \vee \neg p$ d) $p \wedge q \Rightarrow p$

11)Write the converse of the statement: If there is a flood then the crop will be destroyed

Problems

1) Let p , q and r be the propositions . P: you have the free.
Q: you miss the final examination.
R: you pass the course.

Write the following proposition into statement form.

i) P → Q , ii) □ P → R , iii) Q → □ R , iv) P V Q V R v) (P → □R) V (Q → □R)

2) Construct a truth table for each of there (easy) compound statements.

i) (p → q) □ (□ p → q) , ii) p → (□ q v r)

3) Write the negation of the following statements.

i) Jan will take a job in industry of go to graduate school.

ii) James will bicycle or run tomorrow.

iii) If the processor is fast then the printer is slow.

4) Use De morgan’s laws to write the negation of each statements.

i) I want a car and a worth cycle .

ii) My cat stays outside or it makes a mess.

iii) You study or you don’t get a good grade.

5) Let P(x) denote the statement . " x is a professional athlete" and let Q(x) denote the

statement " x plays soccer " . The domain is the let of all people. Write each of the following proposition in English.

i) $\forall x \, (P(x) \rightarrow Q(x))$

ii) $\exists x \, (P(x) \wedge Q(x))$

iii) $\forall x \, (P(x) \vee Q(x))$

SECTION-B

SUBJECTIVE QUESTIONS

1) Consider the following statements

P: Good mobile phones are not cheap

Q : Cheap mobile phones are not good.

L : P implies Q

M : Q implies P

N : P is equivalent to Q

Which of the following about L , M and N is correct .

a) only L is true   b) only M is correct

c) only N is true   d) L, M and N are true.

2) Write the symbolic form for the following statements
i)All animals are cats
ii)Some men are clever

3) Write the logical formulae to represent the statement ?

' Gold and Silver ornaments are precious '

The following notations are used

G(x) : x is a gold ornament

S(x) : x is a silver ornament

O(x) : x is a precious

4) What is the logical translation of the statement not all rainy days are cold is

$\exists d(rainyday \wedge \neg cold(d))$

5) What is the first order calculus statement equivalent to

Tigers and lions attack if they are hungry or threatened.

6) Change the sentence ‘ If I do not have car or I donot wear good dress then I am not a

Millionaire ‘ into symbolic form.

7) If A={ 1,2,3,4,5} be the universal set , determine the truth value of statement .

$(\forall x \in A)(x + 2 < 10)$

8) Negate the statement , some students are 26 or older .

9) Obtain the pdnf for (pvq)

10) Symbolise using quantifiers : Every even number is divisible by 2.

Descriptive Questions

1. Construct the truth table for the given statement formula :

   $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$
2. Identify the valid argument from

   I_1 : If it rains then the cricket match will not be played

   The cricket match played

   Inference : there was no rain.

   I_2 : If it rains then the cricket match willnot be played

   It did not rain.

   Inference : The cricket match was played.
3. Write PDNF of the statement formula : P□(P□ □ Q)
4. Show that $p \rightarrow q \Leftrightarrow \sim p \ V \ q$
5. Obtained the principal conjunctive normal form of $p \vee q \vee (\neg p \wedge \neg q \wedge r)$
6. Use truth table to verify the following logical equivalence $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$
7. Obtain Disjunctive normal form [D.N.F] of $p \wedge (p \rightarrow q)$
8. Obtain principal conjunctive normal form [PCNF] of $[(PVQ) \wedge \sim R] \leftrightarrow Q$.

9. Establish the validity of the argument $p \to q, q \to r, p \Rightarrow r$

C. Questions testing the analyzing / evaluating ability of students

1. Prove that the following argument is valid.
   If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Rochelle did not get the supervisor's position or she did not work hard.

@@@

UNIT-V

Course Objectives: The objective of the course is

- to provide an emphasis on the differences and similarities between fuzzy logic and crisp logic fundamentals.

Syllabus: some more connectives,fuzzy logic,fuzzy groups and fuzzy rings, Homomorphic image and pre-image of subgroupoid

Course Outcomes: At the end of the course students should be able to

- Distinguish between crisp logic and fuzzy logic at conceptual level.
- Demonstrate the idea of fuzzy subgroupoid and fuzzy subring

Learning Material

Introduction: some connectives have already learnt in previous section. we have seen that all connectives are not essential for describing statements for any formula there exist an equivalent formula which can be expressed with the help of connectives belonging to one of the functionally complete sets. Inspite of this fact we define more connectives because some of the formulas can be expressed in simpler forms by using these connectives we now discuss the connectives called exclusive or ,NAND,NOR.

The truth table of above connectives is shown below

P	Q	**P$\uparrow Q$=7(PQ)** (P NAND Q)	$P\downarrow$**Q=7(P□Q)** (P NOR Q)	**P∇Q or P$\oplus Q$** (P exclusive or Q)
T	T	F	F	F
T	F	T	F	T
F	T	T	F	T
F	F	T	T	F

Note:1)The connectives NAND and NOR are dual to each other.

2) To find dual formula which involves NAND and NOR we have to interchange NAND and NOR in addition to other changes mentioned earlier.

Fuzzy logic: The truth value of any proposition is precisely either yes or no and there is no other answer between yes and no but human thought process are imprecise and which allows partial truth lying between yes or no.There are also problems in the real world which are multi valued.

Logical connectives in fuzzy logic:

The logical connectives in fuzzy logic are negation,disjunction,conjunction& implication. Let $\underline{P}$ and $\underline{Q}$ be any two fuzzy propositions be defined for fuzzy sets $\underline{A}$ and $\underline{B}$ respectively and $\underline{P}^1$ and $\underline{Q}^1$ be their negations then

Negation:$T(\underline{P}^1) = 1 - T(\underline{P})$

Disjunction: $T(\underline{P} \vee \underline{Q}) = \max(T(\underline{P}), T(\underline{Q}))$

Conjunction:$T(\underline{P} \wedge \underline{Q}) = \min(T(\underline{P}), T(\underline{Q}))$

Implication:$t(\underline{P} \to \underline{Q}) = T(P^1 \wedge \underline{Q}) = \max(T(P^1), T(\underline{Q}))$

Fuzzy groups and fuzzy rings:

Fuzzy subgroupoid: Let(S, .) be a groupoid (S is a set closed under binary operation '.').

A fuzzy set $\mu : s \to [0,1]$ is called fuzzy subgroupoid of S if $\mu(x,y) \geq \min(\mu(x), \mu(y))$.

Fuzzy subgroup: If G is a group. $\mu : G \to [0,1]$ is called a fuzzy subgrouop of G iff

1) $\mu(x,y) \geq \min(\mu(x), \mu(y)) \forall x, y \in G$

2) $\mu(x^{-1}) \geq \mu(x) \forall x \in G$

Lattice of fuzzy subgroups:The set of all fuzzy subsets of a subgroupoid S is a completelattice L with respect to ordering ≤ defined by fuzzy set inclusion i.e., $\mu_1(x) \leq \mu_2(x) \forall x \in s$ .The least and greatest elements of L are the constant functions $0=1_\theta$ and $1=1_S$ since θ and S are subgroupoids ,0 and 1 are fuzzy subgroupoids.

Homomorphic image and pre-image of subgroupoid:

Homomorphism:Consider two subgroupoids S and T. A mapping f from S to T is said to be a homomorphism if f(xy)=f(x)f(y) $\forall x, y \in S$

Note: **f(S)** is a subgroupoid of T.

Sup property:A fuzzy subset μ of S is said to have a suppropertuy if for any subset $T \subseteq S$ there exist $t_0 \in T$ such that $\mu(t_0) = \sup_{t \in T} \mu(t)$.

Note:A homomorphic image of a fuzzy subgroupoid with sup property is a fuzzy subgroupoid.

Fuzzy subgroup: If G is a group.$\mu: G \to [0,1]$ is called a fuzzy subgroup of G iff 1) $\mu(x, y) \geq \min(\mu(x), \mu(y)) \forall x, y \in G$.

2) $\mu(x^{-1}) \geq \mu(x) \forall x \in G$

Note:1) The characteristic function 1_T is a fuzzy subgroup iff T is a subgroup of G.

2)The intersection of any collection of fuzzy subgroupos is a fuzzy subgroup.

3) The fuzzy subgroup generated by characteristic function of a set is just the characteristic function of the subgroup generated by the set i.e., $(1_A) = 1_{(A)}$

4) Let μ be a fuzzy subgroup of G then $\mu(x^{-1}) = \mu(x)$ and $\mu(x) \leq \mu(e) \forall x \in G$ where $e \in G$ is identity.

5) $G_\mu = \{x \in G \,/\, \mu(X) = \mu(e)\}$ is a subgr oup.

6) $\mu(xy^{-1}) = \mu(e) \Rightarrow \mu(x) = \mu(y)$.

7) μ *is* a constant on each coset of G_μ.

8) μ is a fuzzy subgroup of a group S iff $\mu\left(x, y^{-1}\right) \geq \min(\mu(x), \mu(y)) \forall x, y \in S$.

9) A group cannot be the union of two proper subgroups.

10) A homomorphic image of a fuzzy subgroup is a fuzzy subgroup under sup property and pre image of every fuzzy subgroup is a fuzzy subgroup.

Assignment-Cum-Tutorial Questions

SECTION-A

I. Objective Questions

1. $P \uparrow Q \equiv$ ____________
2. In fuzzy logic the truth value of a proposition lies between _____ and ______
3. The intersection of any set of fuzzy subgroupoids is a ________
4. $P \downarrow Q \equiv$ ____________
5. A mapping f from S to T is a homomorphism if ----------------------
6. State supproperty.
7. A homomorphic image of a fuzzy subgroupoid with supproperty is a -------------
8. If $T(\underline{P}) = .1$ and $T(\underline{Q}) = 0.25$ then $T(\underline{P} \cap \underline{Q}) =$ _______
9. If $T(\underline{P}) = 1$ and $T(\underline{Q}) = 0.25$ then $T(\underline{P} \cup \underline{Q}) =$ _______
10. If $T(\underline{P}) = .9$ and $T(\underline{Q}) = 0.67$ then $T(\underline{P} \leftrightarrow \underline{Q}) =$ _______
11. If $T(\underline{P}) = .89$ and $T(\underline{Q}) = 0.25$ then $T(\underline{P} \rightarrow \underline{Q}) =$ _______
12. If $T(\underline{P}) = .65$ then its negation is-----------------------
13. If $T(\underline{P}) = 0.25$ then $T(\underline{P'}) =$ ______

    (a) 0.75 (b) 0 (c) 1 (d) None.
14. Is the connective NOR is

    (a) Associative (b) not associative (c) Not commutative (d) none.
15. Is $P \nabla Q \equiv \urcorner P \vee Q$

    (a) Yes (b) no (c) cannot be said (d) none.
16. If $T(\underline{P}) = 1$ and $T(\underline{Q}) = 0.25$ then $T(\underline{P} \cap \underline{Q}) =$ _______

    (a) 0.25 (b) 0.75 (c) 1 (d) 0
17. In the above question find $T(\underline{P} \rightarrow \underline{Q})$ is ______

    (a) 0 (b) 1 (c) 0.25 (d) 0.75
18. use data in que.no. 4 to find $T(\underline{P} \leftrightarrow \underline{Q})$

    (a) 0 (b) 1 (c) 0.25 (d) none
19. The dual of $(P \downarrow Q) \downarrow (Q \downarrow R)$ is -----------

    (a) $(P \downarrow Q) \downarrow (Q \downarrow R)$(b) $(P \uparrow Q) \uparrow (Q \uparrow R)$ (c) $Q \uparrow P$ (d) none
20. If $T(\underline{P}) = 1$ and $T(\underline{Q}) = 0$ then $T(\underline{P} \cap \underline{Q}) =$ _______

    (b) 0.25 (b) 0.75 (c) 1 (d) 0
21. Is the connective NAND is

    (a)Associative (b) not associative (c) Not commutative (d) both associative and commutative

22. $T(\underline{P'})$ = -----------------

(a) $T(\underline{P})$ (b)p (c)1- $T(\underline{P})$ (d) none

Descriptive Questions:

1. Write short notes on fuzzy logic.
2. Show that the connective NAND is commutative but not associative.
3. Explain the connective "Exclusive OR" with an example.
4. Convert $P\ (Q \Leftrightarrow R)\, V\, (R \Leftrightarrow P)$ into its equivalent formula which does not involve biconditional and conditional.
5. Show that the connective NOR is commutative but not associative.
6. Explain fuzzy connectives with an example?
7. What is the dual of $P \uparrow ((Q \Leftrightarrow R)\, V\, (R \Leftrightarrow P))$.
8. State fuzzy subgroupoid?
9. Define homomorphism?
10. State fuzzy subgroup?

Problems

1. Show that a homomorphic preimage of a fuzzy subgroupoid of T is a fuzzy subgroupoid of S.
2. Show that the connective ↓ in the following formulae is functionally complete (a) P ↓ P (b) $(P \downarrow Q) \downarrow (Q \downarrow R)$
3. Let $\underline{P}$: Mary is efficient, $\underline{Q}$: Ram ia efficient and $T(\underline{P})$ = 0.8, If $T(\underline{Q})$ = 0.65 find out (a) $T(\underline{P'})$ (b) $T(\underline{P} \wedge \underline{Q})$ (c) $T(\underline{P} \rightarrow \underline{Q})$
4. Show that the connective ↑ in the following formulae is functionally complete (a) P ↑ P (b) $\left(P \uparrow Q\right) \uparrow (Q \uparrow R)$
5. Write a short note on fuzzy logic.
6. Prove that the intersection of family of fuzzy subgroups is a fuzzy subgroup
7. Give an example of a fuzzy proposition and explain it?
8. List the rules of fuzzy approximate reasoning?