GUDLAVALLERU ENGINEERING COLLEGE
(An Autonomous Institute with Permanent Affiliation to JNTUK, Kakinada)
Seshadri Rao Knowledge Village, Gudlavalleru – 521 356.

Department of Computer Science Engineering

HANDOUT

on

BASIC ELECTRICAL ENIGINEERING

HANDOUT ON BASIC ELECTRICAL ENGINEERING

Class & Sem. :I B.Tech – I Semester Year : 2017-18
Branch : CSE Credits : 3

BRIEF HISTORY AND SCOPE OF THE SUBJECT

Electrical engineering is a field of engineering that generally deals with the study and application of electricity, electronics, and electromagnetism. This field first became an identifiable occupation in the latter half of the 19th century after commercialization of the electric telegraph, the telephone, and electric power distribution and use.

Electrical engineering has now subdivided into a wide range of subfields including electronics, digital computers, power engineering, telecommunications, control systems, radio-frequency engineering, signal processing, instrumentation, and microelectronics. The subject of electronic engineering is often treated as its own subfield but it intersects with all the other subfields, including the power electronics of power engineering.

PRE-REQUISITES:

- The student must be familiar with logical thinking.
- The student should have knowledge in physics, electrostatics and mathematics.

COURSE OBJECTIVES:

- To introduce the basics electrical circuits and network theorems.
- To develop an understanding of DC machines and AC machines.

COURSE OUT COMES:

Student will be able to

- Describe the basics of electrical concepts like voltage, current, power and energy
- Analyze an electrical circuit by using various laws.
- Apply the knowledge of network theorems in simplifying electrical networks.
- Demonstrate principles of various D.C. machines.
- Select appropriate DC/AC machine for real time applications.

PROGRAM OUTCOMES:

The **graduate attributes** defined by the NBA are the following:

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization for the solution of complex engineering problems.
2. Problem analysis: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for public health and safety, and cultural, societal, and environmental considerations.
4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of t h e information to provide valid conclusions.
5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools, including prediction and modeling to complex engineering activities, with an understanding of the limitations.
6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and

demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with the society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognizes the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PRESCRIBED TEXT BOOKS:

1. Basic Electrical Engineering - By M.S.Naidu and S. Kamakshiah TMH.
2. Basic Electrical Engineering By T.K.Nagasarkar and M.S. Sukhija Oxford University Press.
3. Electrical and Electronic Technology by hughes Pearson Education.

REFERENCES BOOKS:

1. Theory and Problems of Basic Electrical Engineering by D.P.Kothari & I.J. Nagr ath PHI.
2. Principles of Electrical Engineering by V.K Mehta, S.Chand Publications.
3. Essentials of Electrical and Computer Engineering by David V. Kerns, JR. J. David Irwin Pearson.
4. Basic Electrical Engineering by Vincent Del toro, PHI

COURSES AVAILABLE IN VIDEO LIBRARY:

IIT:

1. Circuit Theory - Prof.S.C.Datta Roy (No. of Video Courses – 51)
2. Networks, Signals and Systems – Prof T.K.Basu (No. of Video Courses – 36)
3. Networks and Systems – Prof.V.G.K.Murti (No. of Video Courses – 50)
4. Special Electrical motors – Prof.K.V.Ratnam (No. of Video Courses – 37)
5. Basic electric circuits - Nagendra Krishnapura (No. of Video Courses – 9)

6. Theory of Electrical Machines -S.N. Bhadra (No. of Video Courses – 42)
7. Basic Electrical Technology – Dr.L.Umanand (No. of Video Courses – 40)

NPTEL :

Basic Electrical Circuits - Nagendra Krishnapura

Fundamentals of Electrical Engineering - Debapriya Das

SONET :

Circuit Theory – N.C. Jagan (Number of Video Courses - 36)

LECTURE SCHEDULE / LESSON PLAN:

Lecture No.	Topics	No. of Periods	
		Lecture	Tutorial
	UNIT-I : ELECTRICAL CIRCUITS		
1	Basic definitions	1	4
2	Types of elements	1	
3	Ohm's Law and its limitations	1	
4	Kirchhoff's laws	1	
5	Simple problems on Kirchhoff's laws	1	
6	Mesh analysis	1	
7	Simple problems on Mesh analysis	2	
8	Nodal analysis	1	
9	Simple problems on Nodal analysis	2	
	UNIT-II : NETWORK THEOREMS		
1	Superposition theorem	1	4
2	Simple problems on Superposition theorem	2	
3	Reciprocity theorem	1	
4	Simple problems on Reciprocity theorem	1	
5	Thevenin's theorem	1	
6	Simple problems on Thevenin's theorem	2	
7	Maximum power transfer theorem	1	
8	Simple problems on Maximum power transfer theorem	1	
	UNIT-III : DC Machines		
1	Construction of DC Generator	2	4
2	Principle of operation of DC Generator	1	
3	Emf equation of DC Generator	1	
4	Simple problems on Emf equation	1	

5	Losses in DC Generator	1	
6	Applications of DC Generator	1	
7	Types of DC Generator	1	
8	Simple problems	2	
UNIT-IV: DC Motors			
1	Construction of DC Motor	1	4
2	Principle of operation of dc motors	2	
3	Torque equation	1	
4	Simple problems on torque equation	1	
5	Losses in DC Motor	1	
6	Types of DC Motors	1	
7	Simple problems	2	
UNIT-V: Transformers			
1	Introduction and Principles of operation of Transformer	2	4
2	Constructional details	1	
3	Losses in transformer	1	
4	O.C and S.C tests	2	
5	Simple Problems on O.C and S.C tests	1	
6	Efficiency and Regulation calculations	2	
7	Numerical problems	2	
UNIT-VI: Three Phase Induction Motors			
1	Construction of induction machine	1	4
2	principle of operation induction machine	1	
3	Slip, rotor frequency	1	
4	Simple Problems on Slip and rotor frequency	2	
5	Torque equation	2	
6	Simple Problems on Torque equation	1	
7	Torque – Slip characteristics	1	
	Total	**60**	**24**

SEMINARS/GROUP DISCUSSION TOPICS:

1. Network reduction techniques
2. Faraday's laws of electromagnetic induction
3. Basic working principle of AC/DC machines and its applications.
4. Real time applications of special motors.

CONTINUOUS EVALUATION PROCEDURE:

The performance of a student shall be evaluated with a maximum of 100 marks in the subject "BASIC ELECTRICAL ENGINEERING", as per the college Curriculum. The distribution shall be 40 marks for Internal

Evaluation and 60 marks for End-Examination. Out of 40 internal marks – 20 marks are assigned for continuous evaluation in the form of class test, 20 marks for MID term exam.

ATTENDANCE:

As per College Curriculum 75% minimum attendance is required for a student for eligibility to appear the end examinations. But for the students who wish to be strong in any subject they should put in a minimum attendance of 90%.

TUTORIAL / ASSIGNMENT QUESTIONS:

Two tutorials per unit will be given to the students comprising of some innovative problems from various text books.

UNIT -I

ELECTRICAL CIRCUITS

Objectives:

- To introduce the basic elements of Electrical circuits.
- To familiarize students with basic principles of electrical laws and analysis of networks.

Syllabus:

ELECTRICAL CIRCUITS:
Basic definitions, types of elements, Ohm's Law, Kirchhoff's laws, mesh and nodal analysis. (All the above topics are only elementary treatment and simple problems).

Learning Outcomes:
Student will be able to

1. Demonstrate the knowledge and understanding of the fundamental principles of electrical engineering.
2. Identify the basic elements in given circuit
3. Apply the basic principles to solve a given network

UNIT-I

ELECTRICAL CIRCUITS

- The valance electrons which are loosely attached to the nucleus of an atom are called free electrons.
- The flow of free electrons is called as electric current.
- Time rate of change of charge is called as electric current.

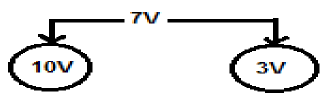
$$i = \frac{dQ}{dt} \text{ Coulomb}/_{sec} \text{ (or)Ampere}$$

If one coulomb charge flows through one section in one second is called as one Ampere current.

- Potential is the work done to move a unit charge through an element.

$$V = \frac{dW}{dQ} \quad \text{Joule}/_{\text{Coulomb}} \text{ (or)Volts}$$

- The difference in the potential of two charged bodies is called as



potential difference.
Units: Volt
- Total work done in electric circuit is called as energy (E).
Units: Joules
- Rate of transfer of energy is called as power (P).

$$P = \frac{dW}{dt}$$

$$P = \frac{dW}{dQ} * \frac{dQ}{dt}$$

$$P = V * I$$

"The rate at which work is done in electric circuit is called as power".

Classification of elements:

1. **Active & passive:**

An element is said to be active, if it is able to deliver the energy to outside world for infinite time, otherwise passive. Example for active elements are sources and passive are R, L, C.

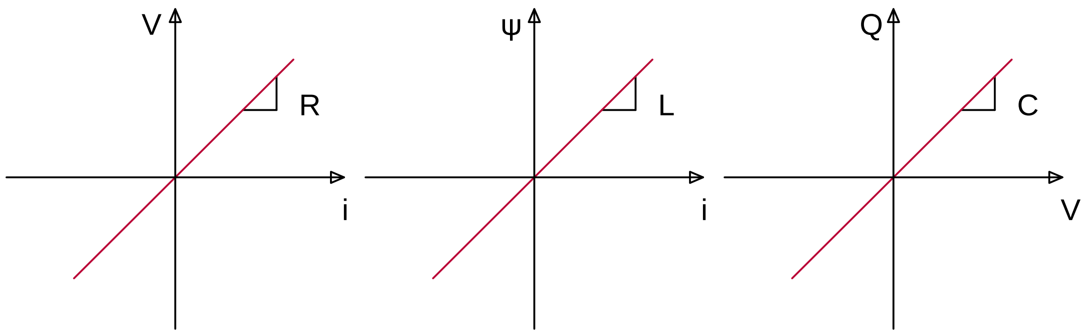
Note:

1. Ohm's Law is not applicable for active elements.

2. If V/I ratio is positive, then it is called as passive element passive elements cannot supply more energy than what it had drawn previously.

2. Linear & Non-linear elements:

If the characteristic of an element is a straight line passing through the origin, it is called as linear element and these characteristics are constant.



Examples:

- Linear elements are R, L, and C.
- Non-Linear elements are Diode, Transistor.

3. Unilateral &Bilateral elements:

If an elements offers same impedance (opposition) for both the directions of flow of current through it is called as bilateral element otherwise it is unilateral element.

Examples:

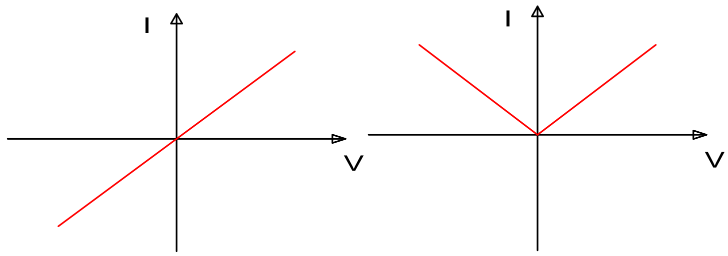
- Bilateral elements are R, L, C.
- Unilateral element is Diode, transistor.

For forward voltage Diode acts as short circuit. i.e. R=0. In reverse Bias it acts as open circuit i.e. R is infinity. So here Diode offers different resistance for different of current. Therefore, it is called as unilateral element.

If V/I characteristics are same in all direction, it is called as Bilateral element.

4. Time variant / invariant:

If the element characteristics are independent of time, it is called as time variant, otherwise time variant.



(i) (ii)

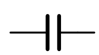
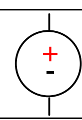
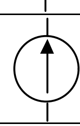
Case (i)

$\frac{V}{I}$ is Positive. ∴ It is passive element, bilateral element, linear element.

Case (ii)

$\frac{V}{I}$ is Positive in one Quadrant and $\frac{V}{I}$ is Negative in other direction. ∴ $\frac{V}{I}$ ratio is not same in both directions. ∴ It is active element, unilateral element, non-linear element.

Basic Symbols

	Symbol / Notations	**Units**
Resistor		Ohm's (Ω)
Inductor		Henry (H)
Capacitor		Farad (F)
Voltage Source		Volts, V
Current Source		Amperes (A)

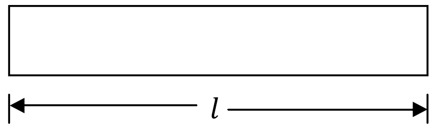
Resistance: (R)

It is a property of a material, which opposes the flow of electric current.

Units: ohm's Ω

Let ‘l’ be the length of the material

A be the cross sectional area of material.



Resistance is directly proportional to length of the material,

$$R \propto l \qquad (1.1)$$

As the area of cross section increases, electron can move freely.

∴Resistance is inversely proportional to the area of cross section.

$$R \propto \frac{1}{A} \qquad (1.2)$$

From (1) & (2)

$$R \propto \frac{l}{A} \qquad\qquad R = \frac{\rho l}{A}$$

$$\rho = \text{Resistivity (or) Specific Resistance}$$

$$\rho = \frac{RA}{l} = \frac{\Omega * m^2}{m} = \Omega - m$$

Reciprocal of resistance is called as conductance. It is denoted by ‘G’

$$G = \frac{1}{R}$$

Units: Mho’s (℧)

Open circuit:

- Open circuit is an element where resistance tends to infinity.
- Current doesn’t flow through open circuit.

Short circuit:

- Short circuit is an element when resistance approaches to zero.
- Potential difference in short circuit is zero.

Ohm’s Law:

“Under constant temperature , current flowing through a conductor is directly proportional to the voltage applied across it”.

Where, R=Resistance of conductor

(1.3)

(1.4)

Conclusion:

From (1.3) & (1.4) power dissipated by resistor remains same.

It is independent of direction of applied voltage (or) current.

Inductance:

“The property of coil that opposes any change in the amount of current flowing through it is called as Inductance”.

Flux linkage depends on the amount of current flowing through the coil.

[L=Inductance of coil]

According to Faraday’s Law

According to Lenz's Law, induced emf should oppose the change in current flow through that coil.

The direction of induced voltage is given by,

$$\text{Energy stored in the inductance (E)} = \int P\,dt$$

$$= \int vi\,dt = \int L\frac{di}{dt}i\,dt = \frac{L}{2}\int (2i)\frac{di}{dt}\,dt$$

$$E = \frac{1}{2}Li^2$$

Properties of inductor:

1. Since it does not allow the sudden change in current through it, it is called as current stiff element.
2. It stores the energy in the form of magnetic field.
3. If the applied voltage is positive, it will start charging and if the applied voltage is negative, it will start discharging.

Capacitor:

Any two conducting surfaces separated by on insulating material (dielectric) is called as capacitor.

Capacitance:

The ability of a capacitor to start charge is known as its capacitance.

Charge stored in capacitor is proportional to applied voltage.

$$\therefore Q \propto V$$

$$Q = CV$$

$$\text{We know that, } i = \frac{dQ}{dt} = \frac{d}{dt}(CV)$$

$$i = C\frac{dV}{dt}$$

Energy stored in capacitor:

Let us consider 'V' voltage is applied across capacitor. At this instant, 'W' joules of work will be done in transferring 1C of charge from one plate to other.

If small charge dq is transferred, then work done is

$$dW = Vdq$$

$$W = \int_0^V CVdq \quad W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}C\left[\frac{q}{C}\right]^2 \quad W = \frac{1}{2}\frac{q^2}{C}$$

Reactance offered by capacitor (X_C):

Let us consider ($V = V_m \sin \omega t$) V volts applied across capacitor.

$$i = C\frac{dV}{dt} \quad i = C\frac{d(V_m \sin \omega t)}{dt}$$

$$i = CV_m\frac{d(\sin \omega t)}{dt} \quad i = CV_m\omega \cos\omega t$$

$$i = V_m \omega C\, j \sin\omega t$$

$$i = \frac{V_m \sin\omega t}{\frac{1}{j\omega C}}$$

$$\boxed{X_C = \frac{1}{j\omega C}}$$

Where, $\omega = 2\pi f$

Properties of capacitor:

1. It doesn't allow the sudden change in voltage. It is called as voltage stiff element.
2. It stores energy in the form of electrostatic field.

For D.C, frequency (f) = 0. i.e $\omega = 0$

$$X_L = j\omega L = j(2\pi f)L = j(2\pi * 0)L = 0$$

For D.C supply opposition offered by inductor is Zero.

i.e. it (Inductor) acts as short circuit.

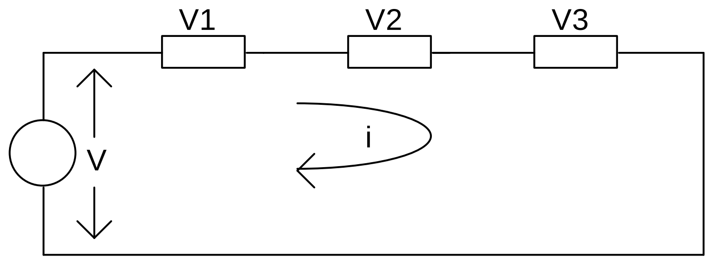
$$X_C = \frac{1}{j\omega C} = \frac{1}{j(0)C} = \infty$$

For D.C, opposition offered by capacitor is Infinity.

i.e. it (capacitor) acts as open circuit.

Kirchhoff's voltage Law: (KVL)

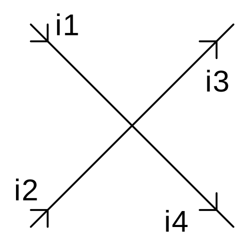
This law is related to emf's and voltage drops in a circuit. It stated as "in an electrical circuit, algebraic sum of all the voltages in a closed path is Zero".



- KVL is independent of nature of element.
- KVL is based on law of conservation of energy.

Kirchhoff's current Law:

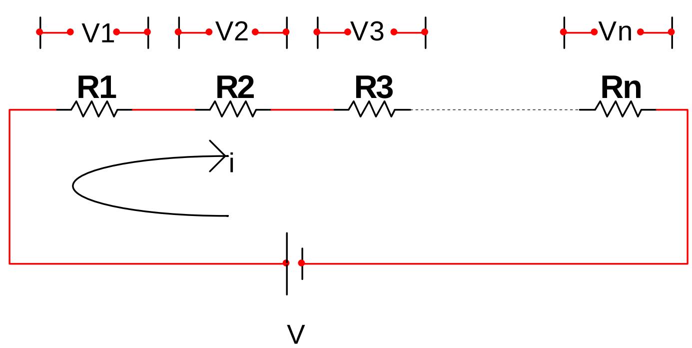
This law is related to current at the junction points a circuit. It is stated as "In a circuit, at node at any instant the algebraic sum of current flowing towards a junction in circuit is Zero".



- According to law of conservation of energy, the net charge at node is Zero.
- KCL is independent of nature of element.

Series R-circuit:

Let us consider 'n' Resistors are connected in series.

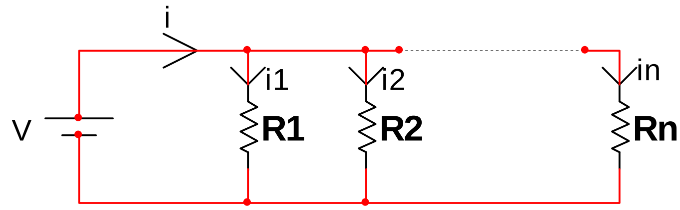


Apply KVL

Note: If 'n' Resistors are in series, then equivalent Resistance will be greater than , , ...

Parallel circuit:

Apply KCL

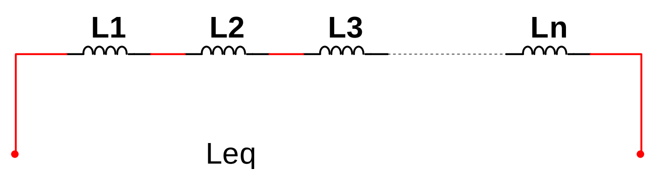


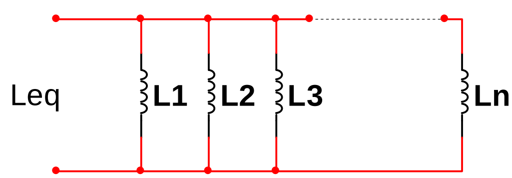
- When 'n' Resistances are in parallel, equivalent Resistance is smaller than all Resistances.

NOTE:

- When 'n' Resistances are in series, the current through all the Resistors are same.
- When 'n' Resistors are in parallel, then voltage across all resistors is same.

Inductive circuits:

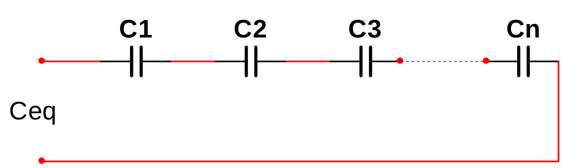




Capacitive circuits:

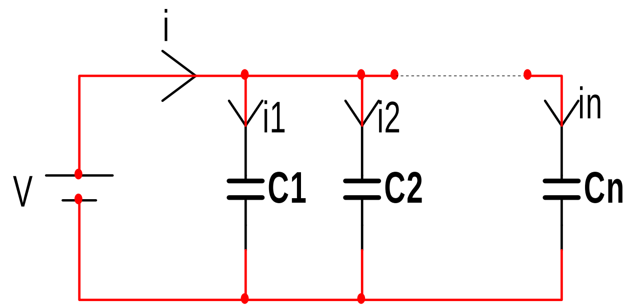
Series circuit:

Apply KVL

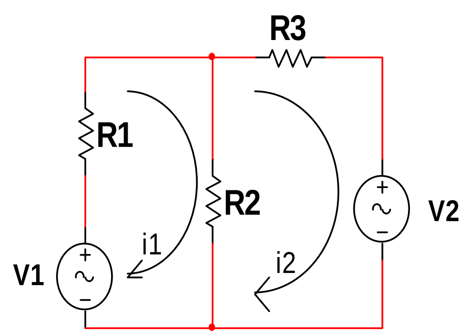


Parallel circuit:

Apply KCL



Mesh analysis (KVL + ohm's Law)



1. Identify the Number of Loops/ Meshes.
2. Assign the currents in each loop.
3. Apply KVL for each mesh and write ohm's law form.
4. Solve the equations and obtain mesh currents.

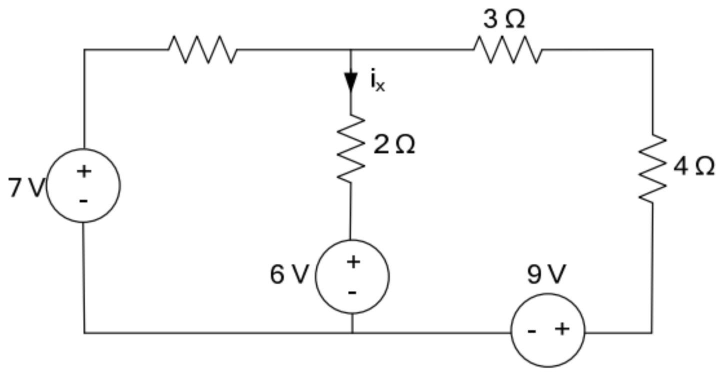
Apply KVL for loop (1)

For loop (2)

Problems on Mesh Analysis

Problems:

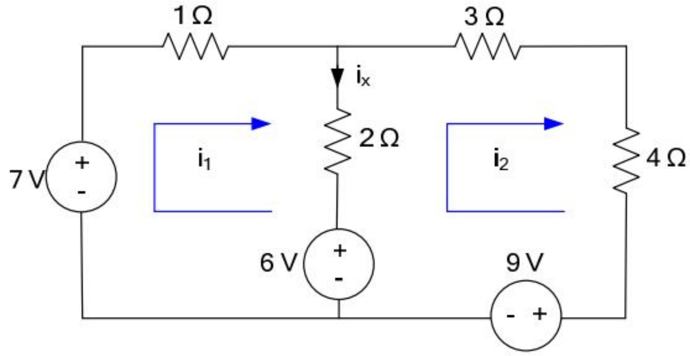
For following circuit find the mesh currents and use the mesh currents to find i_x



Solution

Step #1 Identify all of the meshes.

Step #2 Assign currents to all of the meshes



Step #3 Apply the KVL around each mesh.In this step for each mesh we describe the branch voltages in term of mesh currents.

Apply KVL around mesh # 1

$$-7 + 1*i_1+2*(i_{1-}\ i_2) + 6 = 0$$

$$3\ i_1+2\ i_2=1.....................(1)$$

Apply KVL around mesh # 2

$$3i_2 + 4\ i_2 +9-6+2(i_2- i_1) = 0$$

$$-2\ i_1 + 9\ i_2 = -3............(2)$$

Step #4 Solving question 1 and 2 for the unknown mesh currents (i1 and i2):

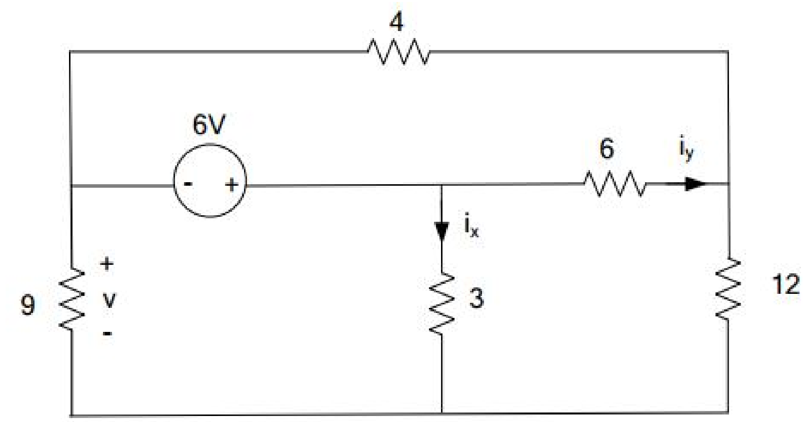
$$i_1 = 0.13A$$

$$i_1 = -0.304A$$

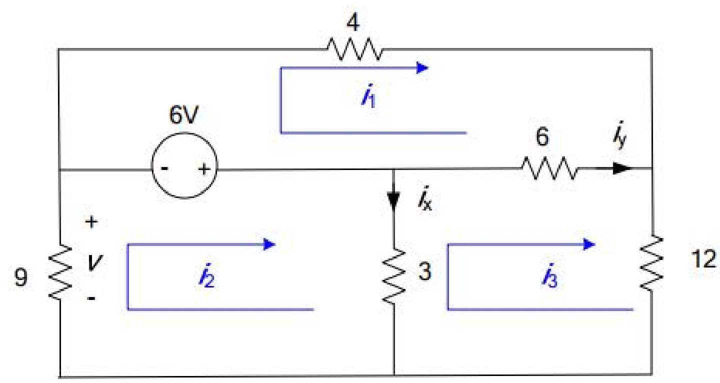
$$i_x = i_1- i_2 = 0.434A$$

Problems:

Write the mesh (loop) equations for the following circuit and then find i_x, i_y and v



Solution:



Apply KVL around mesh # 1

$6+4i_1+6(i_1-i_3) = 0$

$10i_1-6i_3=-6$...............(1)

Apply KVL around mesh # 2

$9i_2-6+3(i_2-i_3) = 0$

$12i_2-3i_3=6$...............(2)

Apply KVL at mesh # 3

$3(i_3-i_2) + 6(i_3-i_1) + 12i_3=0$

$-6i_1-3i_2+21i_3=0$..........(3)

Solve the three equations

$i_1 = - 0.6757$ A

$i_2 = 0.4685$ A

$i_3 = - 0.1261$ A

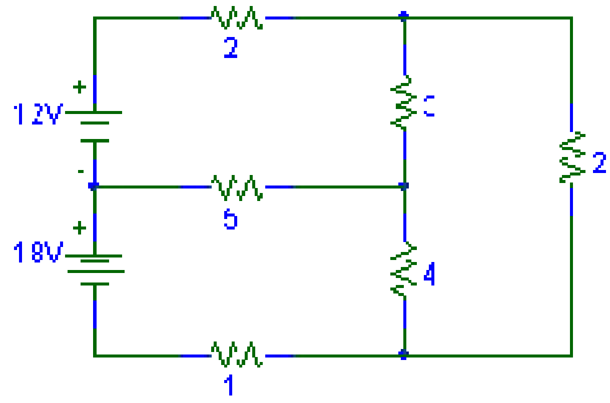
$i_x =i_1-i_2 = -1.1442$ A

$i_y = i_3-i_2= -0.5946$ A

$v = -9i_2 = 4.2165$ V

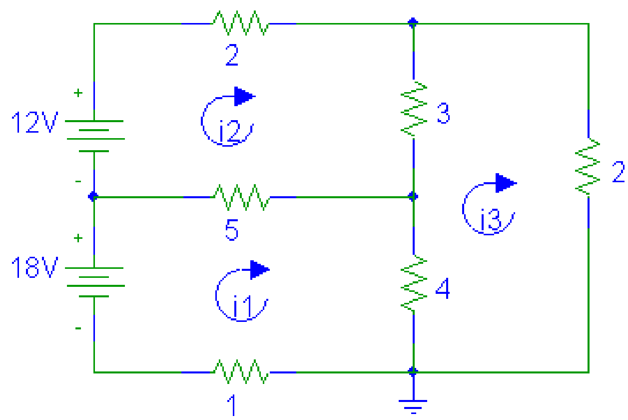
Problem:

Solve for the current through the 5 ohm resistor and the current through the 18V source using Mesh-Current Analysis.



Solution

First, label each mesh (window pane) with a mesh current. For consistency, make each mesh in a clock-wise direction.



Now write KVL equations for each loop.

KVL for i1:

$$-18V + 5(i1-i2) + 4(i1-i3) + 1(i1) = 0$$

then gather terms:

$$10i1 - 5i2 - 4\ i3 - 18V = 0$$

Note that the i1 term is positive, and all other current terms are negative (because they are all clockwise, all other panes will contribute a negative term). Let's do the other two panes with terms gathered up directly (write the total resistance of the loop multiplied by the mesh current that goes through that total resistance):

KVL for i2:

$$-5i1 + 10i2 - 3i3 - 12 = 0$$

KVL for i3:

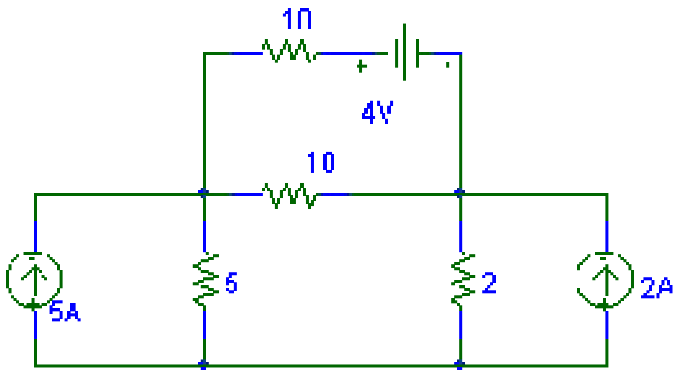
$$-4i1 - 3i2 + 9i3 = 0$$

Now solve the three equations in three unknowns:

i1 = 7.02A
i2 = 6.28A
i3 = 5.21A

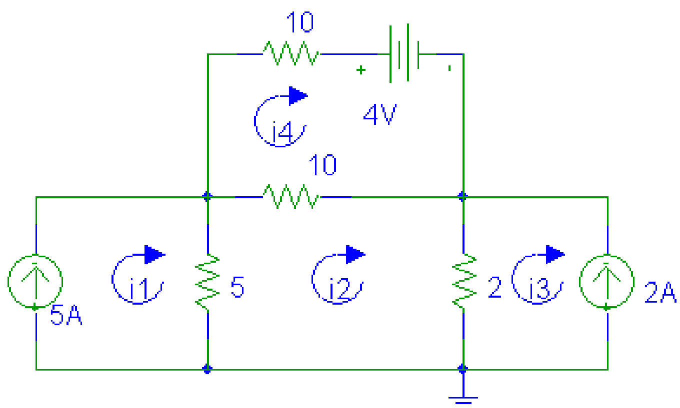
Problem:

Solve for the current through the 5 ohm resistor and the current through the 4V source using Mesh-Current Analysis.

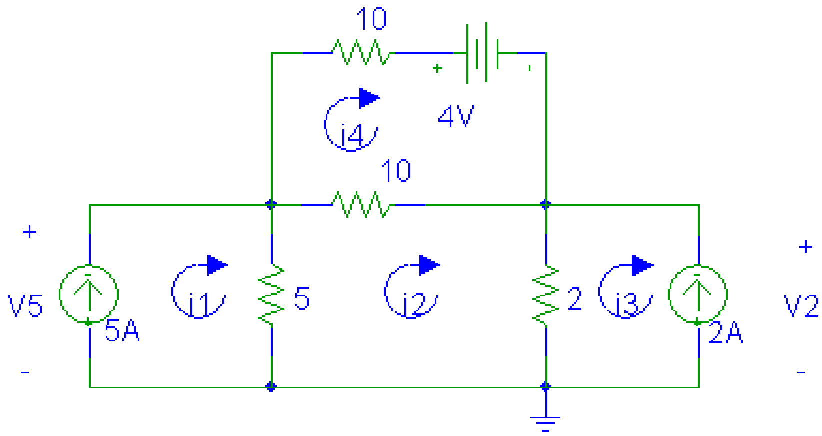


Solution

First, define a mesh current arround each mesh (window pane) of the circuit. Define each one in a clockwise direction.



Now write KVL equations for each loop. On the first loop, we run into a problem. We do not know the voltage across the 5A source. (This is one of the difficulties you can run into when using mesh current.) We add a new unknown to handle this problem. We run into the same problem with the 2A source, so two new unknowns (V5 and V2) are labeled:



Now we are ready to write the KVL equations.

KVL loop 1:

$$-V5 + 5(i1-i2) = 0$$

KVL loop 2:

$$5(i2-i1)+10(i2-i4)+2(i2-i3) = 0$$

or

$$-5i1 + 17i2 - 2i3 = 0$$

KVL loop 3:

$$2(i3-i2) + V2 = 0$$

KVL loop 4:

$$10i4 + 4V + 10(i4-i2) = 0$$

or

$$-10i2 + 20i4 = -4$$

We have two extra unknowns, so we need two more equations. These can be found by examining the loops with the sources:

$$i1= 5A$$

$$i3 = -2A$$

We can then solve for the remaining unknowns:

$$i2 = 1.58A$$

$$i4 = 0.59A$$

We can then find the current through the 5 ohm resistor as:

$$I = i1 - i2 = 3.42A$$

The current through the 4V source is simply i4, or 0.59A.

Nodal Analysis

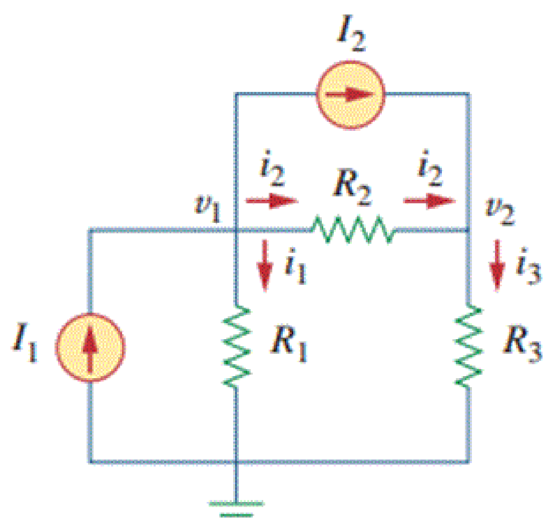
Nodal analysis is a method that provides a general procedure for analyzing circuits using node voltages as the circuit variables. **Nodal Analysis** is also called the **Node-Voltage Method**. **Some Features of Nodal Analysis are as**

- **Nodal Analysis** is based on the application of the Kirchhoff's Current Law (KCL).
- Having 'n' nodes there will be 'n-1' simultaneous equations to solve.
- Solving 'n-1' equations all the nodes voltages can be obtained.
- The number of non reference nodes is equal to the number of Nodal equations that can be obtained.

Solving of Circuit Using Nodal Analysis

Basic Steps Used in Nodal Analysis

1. Select a node as the reference node. Assign voltages V_1, V_2....... V_{n-1} to the remaining nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the non reference nodes.
3. Use Ohm's law to express the branch currents in terms of node voltages.



Node Always assumes that current flows from a higher potential to a lower potential in resistor. Hence, current is expressed as follows

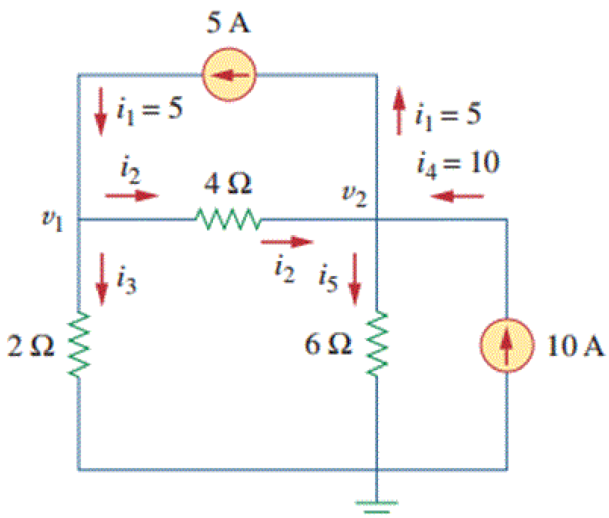
$$I = \frac{V_{high} - V_{low}}{R}$$

IV. After the application of Ohm's Law get the 'n-1' node equations in terms of node voltages and resistances.
V. Solve 'n-1' node equations for the values of node voltages and get the required node Voltages as result.

Nodal Analysis with Current Sources

Nodal analysis with current sources is very easy and it is discussed with a example below.
Example: Calculate Node Voltages in following circuit

In the following circuit we have 3 nodes from which one is reference node and other two are non reference nodes - Node 1 and Node 2.
Step I. Assign the nodes voltages as v_1 and $_2$ and also mark the directions of branch currents with respect to the reference nodes



Step II. Apply KCL to Nodes 1 and 2
KCL at Node 1

$$i_1 = i_2 + i_3$$

KCL at Node 2

$$i_2 + i_4 = i_1 + i_5$$

Step III. Apply Ohm's Law to KCL equations
• Ohm's Law to KCL equation at Node 1

$$i_1 = i_2 + i_3 \;=>\; 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

Simplifying the above equation we get,

$$3v_1 - v_2 = 20............(i)$$

Now ,Ohm's Law to KCL equation at Node 2

$$i_2 + i_4 = i_1 + i_5 \;=>\; \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6}$$

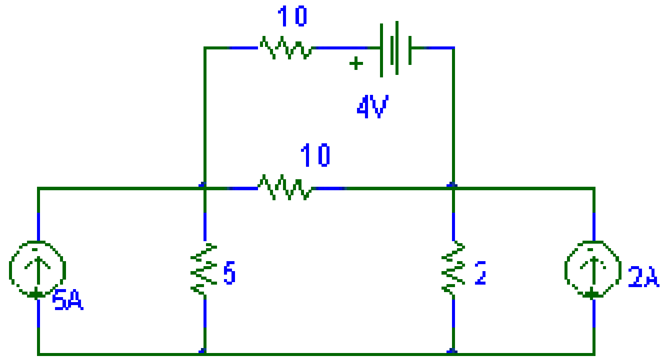
Simplifying the above equation we get

$$-3v_1 + 5v_2 = 60............(ii)$$

Step IV. Now solve the equations (i) and (ii) to get the values of v_1 and v_2 we get node voltages are as $\mathbf{V_1}$ = 13.33 Volts and V_2 **= 20** Volts.

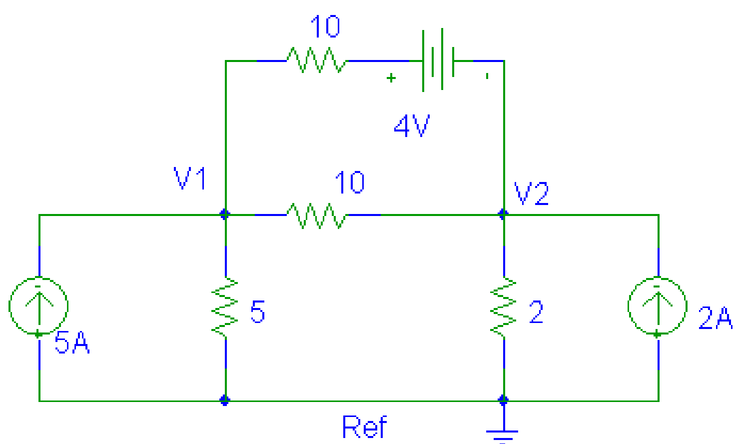
Problem

Solve for the current through the 5 ohm resistor and the current through the 4V source using Node-Voltage Analysis.



Solution

First, select a reference node and label the other nodes. Since each node has the same number of connected branches (4), we'll simply choose the bottom node as the reference. There are only two other nodes, which we will label V1 and V2.



Now write KCL at each node (except the reference):

KCL at V1:

$$-5A + V1/5 + (V1-V2)/10 + [V1-(V2+4)]/10 = 0$$

Note that there are four terms in the equation, one for each branch leaving the node. The terms list the current leaving right, down, left, and up.

KCL at V2:

$$(V2-V1)/10 + V2/2 - 2A + [V2-(V1-4)]/10 = 0$$

Note that there are four terms in the equation, one for each branch leaving the node. The terms list the current leaving right, down, left, and up.

Now gather terms (multiplying through by 10 to clear up the fractions):

$$4V1 - 2V2 = 54$$

$$-2V1 + 7V2 = 16$$

Now solve the set of 2 equations with 2 unknowns.

V1 = 17.08V

V2 = 7.17V

We can now determine the current through the 5 ohm by Ohm's law:

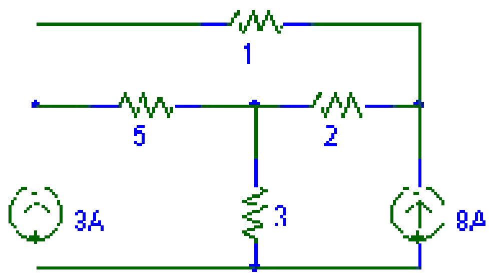
I = V1/5 = 3.41A

The current through
 the 4V source can be found as:

I = [V1-(V2+4)]/10 = 0.59A

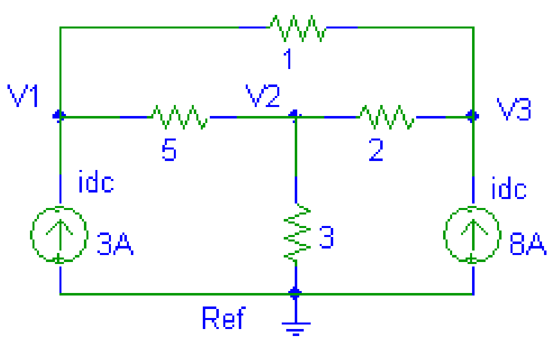
Problem:

Solve for the current through the 5 ohm resistor and the voltage over the 3A source using Node-Voltage Analysis.



Solution

First, select a reference node and label the other nodes. All the nodes have three connected branches, so we will simply choose the bottom node as the reference.



Now write KCL equations for each node except the reference, in terms of the node voltages:

KCL at V1:

-3A + (V1-V2)/5 + (V1-V3)/1 = 0

KCL at V2:

(V2-V1)/5 + V2/3 + (V2-V3)/2 =0

KCL at V3:

(V3-V2)/2 + (V3-V1)/1 - 8A = 0

Now gather terms and clear up the fractions:

6V1 - V2 - 5V3 = 15

-6V1 + 31V2 - 15V3 =0

-2V1 -V2 +3 V3 = 16

Finally, solve the 3 equations in 3 unknowns.

V1 = 48.625V

V2 = 33 V

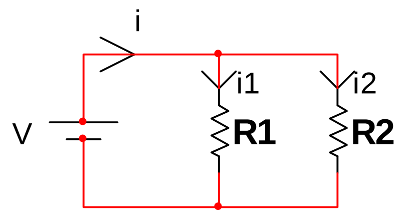
V3 = 48.75V

The current through the 5 ohm resistor can be found by Ohm's law:

I = (V1 - V2)/5 = 3.125A

The voltage over the 3A source is simply V1, or 48.625V

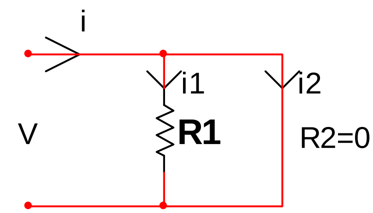
Current division Rule:



Case (i):

$$i_1 = \frac{i * R_2}{R_1 + R_2} = 0$$

$$i_2 = \frac{i * R_1}{R_1 + R_2} = i$$

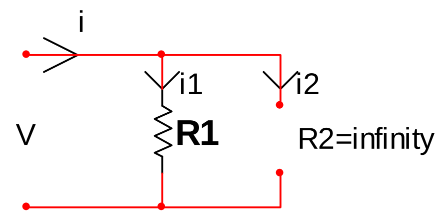


Observation: Current always choose lower Resistance path.

Case (ii)

$$i_1 = \frac{i * R_2}{R_1 + R_2} = i$$

$$i_2 = \frac{i * R_1}{R_1 + R_2} = 0$$



Note: Current will not flow through open circuit.

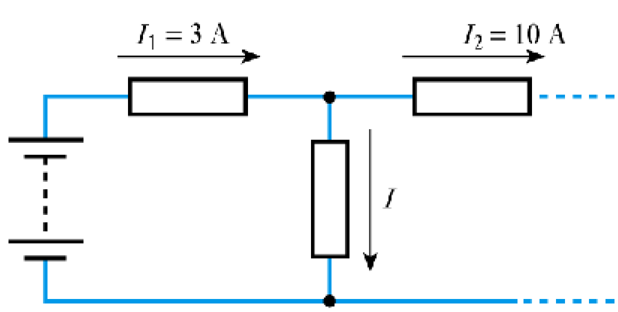
lows brighter

UNIT -I
Assignment Cum Questions
Section - A

Objective Questions

1. Mesh analysis is a combination of [ ]
   a) Ohm's law and KCL **b**) Ohm's law and KVL
   c) KVL and KCL d) None of the above
2. Which of the following element does not allow the sudden change in voltage [ ]
   a) Resistor **b**) Inductor c) Capacitor d) none of these.
3. Ohm's law is applicable when the temperature is [ ]
   a) Positive b) negative c) Variable **d**) Constant
4. Two or more number of current sources can be represent as single current source when they are in [ ]
   a) parallel b) series c) either series or parallel
   d) not possible
5. Short circuit is an element when the resistance approaches [ ]
   a) zero b) infinity c) 1 Ω d) negative value
6. An ideal voltage source should have [ ]
   a) large value of EMF b) small value of EMF
   c) Zero source resistance d) Infinite source resistance
7. Kirchhoff's voltage law is based on law of conservation of [ ]
   a) charge **b**) energy c) momentum d) mass
8. Nodal analysis is a combination of [ ]
   a) Ohm's law and KCL b) Ohm's law and KVL
   c) KVL and KCL d) None
9. Resistivity of a material depends on [ ]
   a) Type of the material b) current c) applied voltage d) none of these
10. The unit of electrical conductivity is [ ]
    a) mho / meter b) mho / sq. m c) ohm / meter d) ohm / sq. m

11. Calculate the magnitude of the current I in the following circuit.[ ]



a) +7 A b) -13 A **c**) -7 A d) +13 A

12. A circuit contains two unequal resistances in parallel [ ]

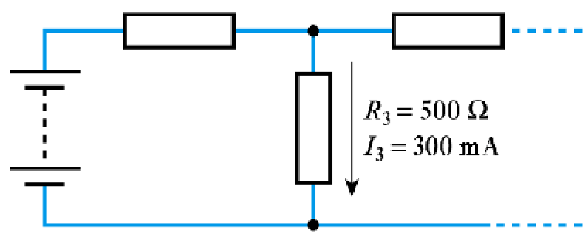
a) current is same in both

b) large current flows in larger resistor

c) Potential difference across each is same

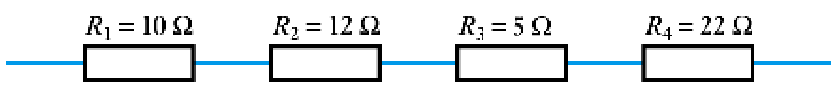
d) smaller resistance has smaller conductance

13. The magnitude of the power dissipated in R_3 in the following circuit[ ]



a) 45 W b) 150 mW c) 45 mW d) 150 W

14. Calculate the effective resistance of the following combination [ ]

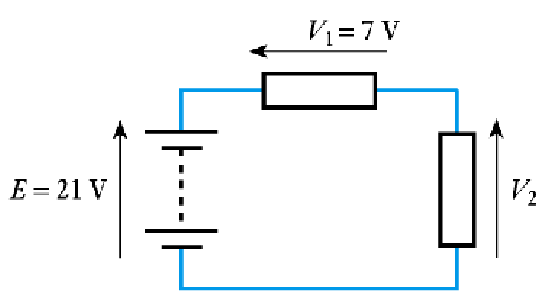


a) 49 ohm b) 39 ohm c) 52 ohm d) 12 ohm

15. The resistance of a 100 W, 200 V lamp is [ ]

a) 100 ohm b)200 ohm **c**)400 ohm d) 1600 ohm.

16. Calculate the magnitude of the voltage V_2 in the following circuit.[ ]

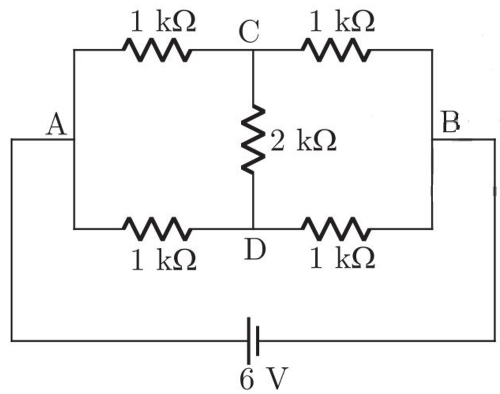


a) 14 V b) -7 V c) 28 V d) -14 V

17. The resistance of a conductor of diameter d and length l is R Ω. If the diameter of the conductor is halved and its length is doubled, the resistance will be [ ]

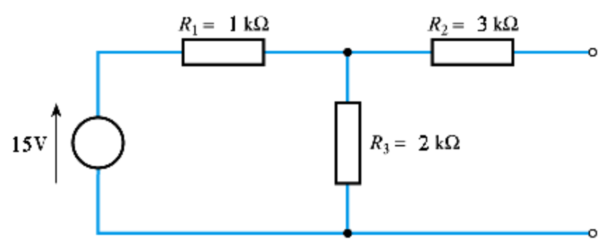
a) R Ω b) 2R Ω c) 4R Ω **d**) 8R Ω

18. The current through the 2 kΩ resistance in the circuit shown is[ ]



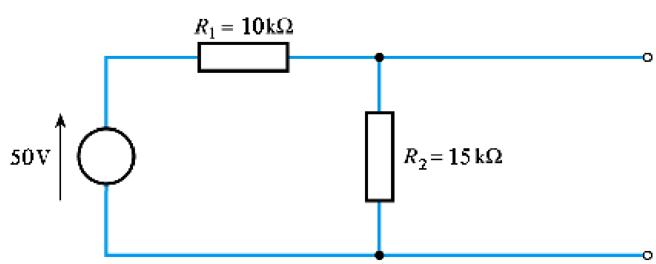
(**A**) 0 mA (B) 1 mA (C) 2 mA (D) 6 mA

19. The open-circuit output voltage of the following circuit [ ]



a) 5 V b) 6.82 V c) 8.18 V **d**) 10 V

20 Determine the short-circuit current of the following circuit [ ]



a) 2 mA b) 3.33 mA **c**) 5 mA d) 6.67 mA

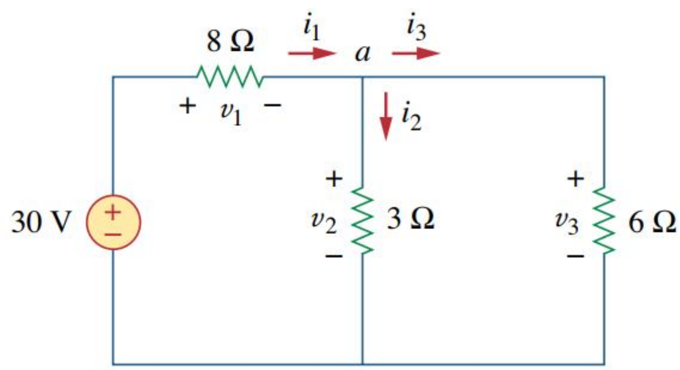
Section – B

Descriptive Questions

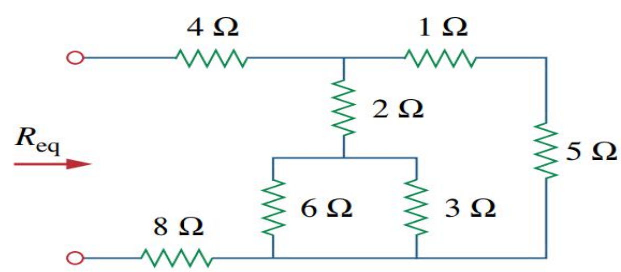
1) What are the various types of sources? Briefly discuss them with necessary circuit diagram and characteristics.

2) Explain are the classifications of network elements and explain.

3) State and expalin Kirchoff's current law andKirchoff's voltage law.
4) State ohm's law with its limitations.
5) If N resistors are connected in parallel of value RΩ, Find the equivalent resistance.
6) Explain the procedure to solve a network using Mesh analysis.
7) Explain the procedure to solve a network using Nodal analysis.
8) Calculate currents and voltages in the circuit shown below by using ohm's law and Kirchhoff's laws



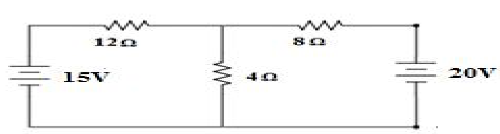
9) Determine Req for the circuit shown below



10) A circuit consisting of three resistances 2Ω, 18Ω and 36Ω respectively

joined in parallel is connected in series with a fourth resistance. The whole circuit is applied with 60V and it is found that the power dissipated in the 12Ω resistor is 36 W. Determine the value of the fourth resistance and the total power dissipated in the circuit.

11) For the circuit shown below calculate current through 12Ω resistance using nodal analysis



12) Determine current through 2Ω resistance for the circuit shown in

figure1 using mesh analysis?

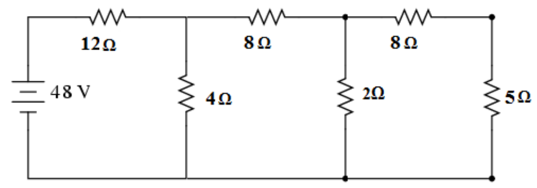
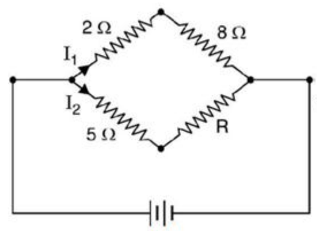
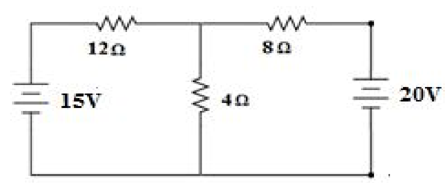


Figure 1

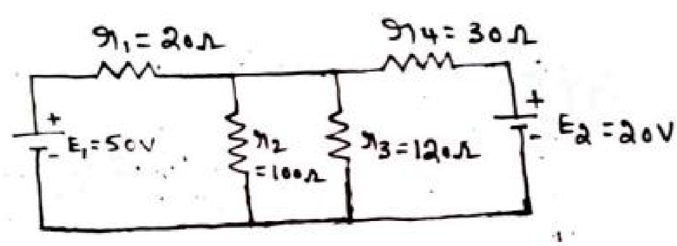
13) Determine the currents in each branch of the below network, if the total current is 2.25A. Also determine the value of R



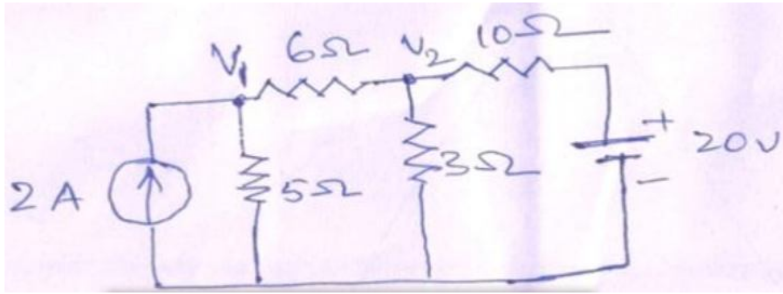
14) Calculate current flowing through 4Ω Resistor for the given circuit by applying Mesh analysis



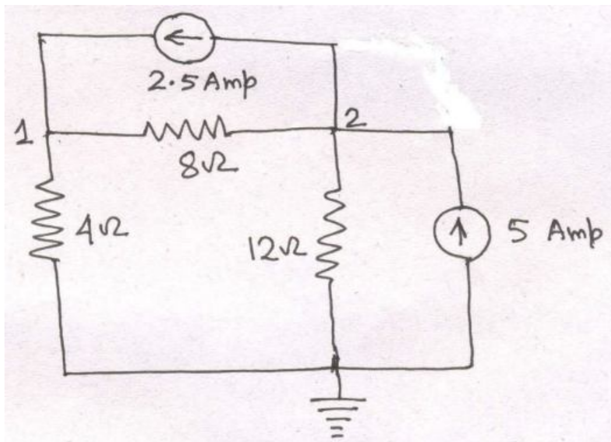
15) Using Mesh and Nodal analysis, find the current through r2 in the circuit shown in below figure.



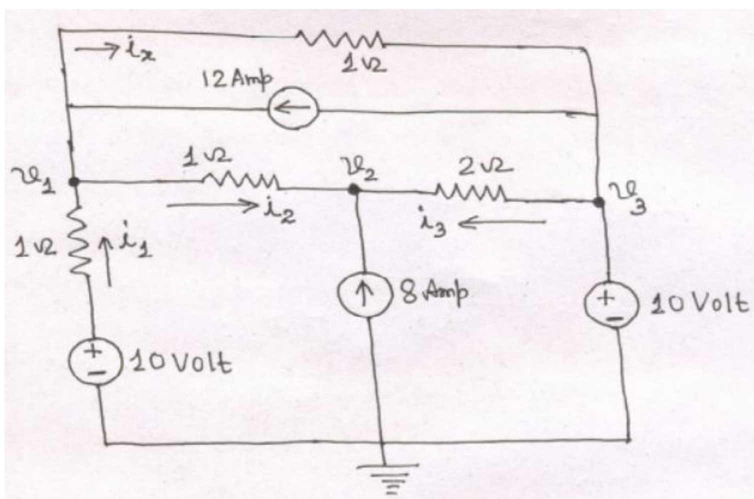
16) Apply nodal analysis to determine node voltages v1 and v2 for the circuit shown below



17) Determine node voltages in the circuits shown below



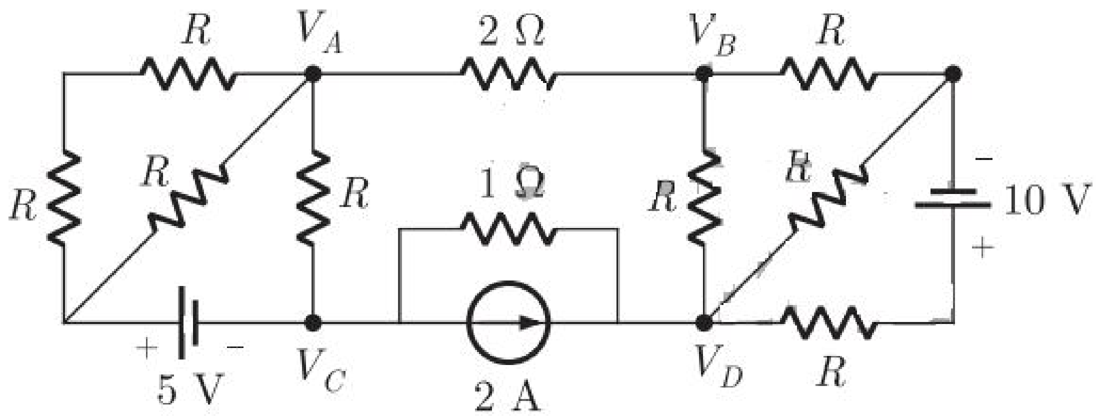
18) Determine node voltages V_1, V_2,V_3 in the circuit shown below



SECTION - C

Gate and IES Questions:

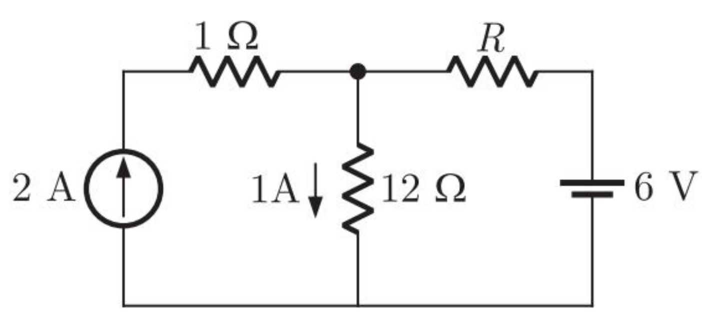
1. If V_A-V_B = 6 V then V_C-V_D = 6 V GATE-2012



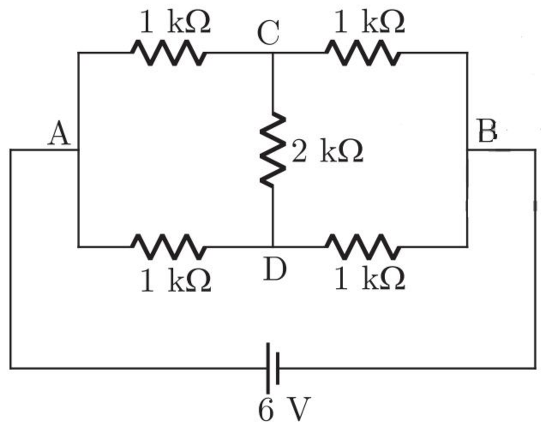
(**A**) −5 V (B) 2 V (C) 3 V (D) 6 V

2. If the 12 Ω resistor draws a current of 1 A as shown in the figure, the value of resistance R is GATE-2010

(A) 4 Ω (**B**) 6 Ω (C) 8 Ω (D) 18 Ω



4. The current through the 2 kΩ resistance in the circuit shown is GATE- 2009

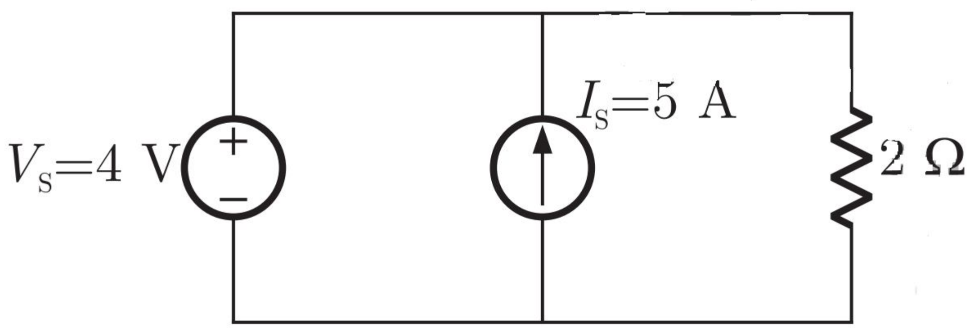


(**A**) 0 mA (B) 1 mA (C) 2 mA (D) 6 mA

4. How many 200 W/220 V incandescent lamps connected in series would consume the same total power as a single 100 W/220 V incandescent lamp? GATE- 2009

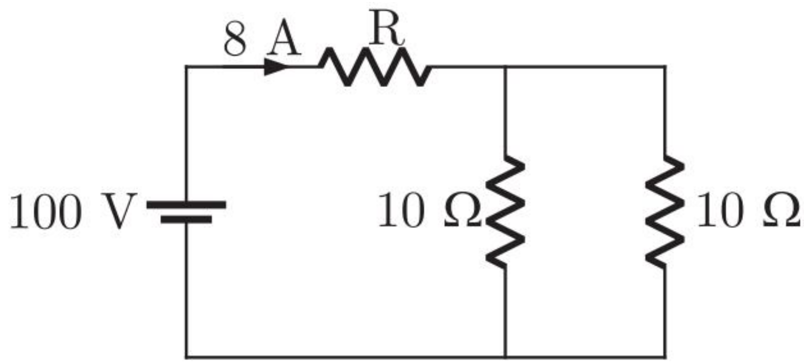
(A) not possible (B) 4 (C) 3 (**D**) 2

5. For the circuit shown, find out the current flowing through the 2 Ω resistance. Also identify the changes to be made to double the current through the 2 Ω resistance. GATE- 2009



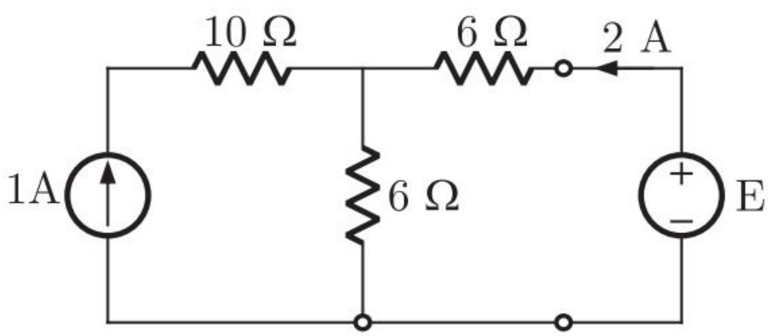
(A) 5A; Put V_S = 30V (B) 2A; Put V_S = 8V
(C) 5A; Put I_S = 10A (D) 7A; Put I_S = 12A

6. In the figure given below the value of R is GATE- 2005



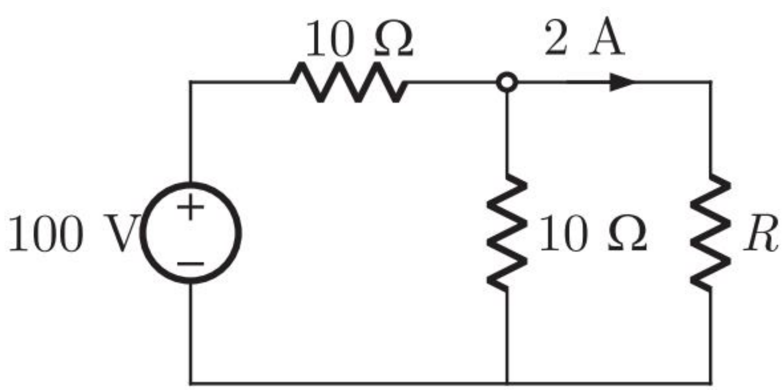
(A) 2.5 Ω (B) 5.0 Ω (**C**) 7.5 Ω (D) 10.0 Ω

7. In figure, the value of the source voltage is GATE- 2004



(A) 12 V (B) 24 V (**C**) 30 V (D) 44 V

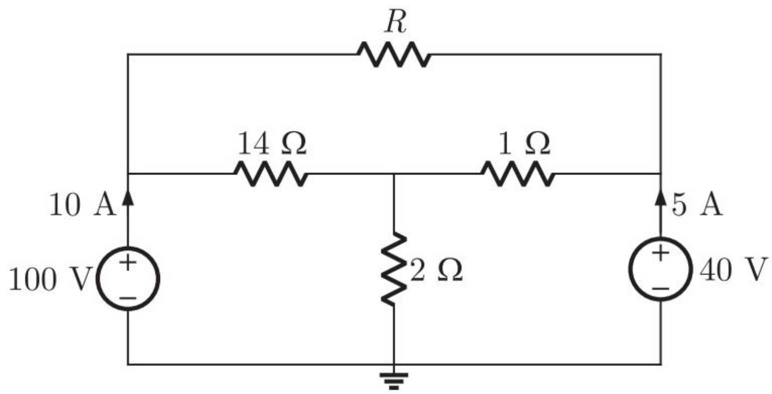
8. In figure, the value of resistance R in Ω is GATE- 2004



(A) 10 (**B**) 20 (C) 30 (D) 40

9. In figure, the value of R is GATE- 2003

(A) 10 Ω (B) 18 Ω (C) 24 Ω (**D**) 12 Ω



10. Two incandescent light bulbs of 40 W and 60 W rating are connected in series across the mains. Then GATE- 2001

(A) the bulbs together consume 100 W

(B) the bulbs together consume 50 W

(C) the 60 W bulb glows brighter

(**D**) the 40 bulb glows brighter

UNIT II
NETWORK THEOREMS

Objectives:

- To impart some basic knowledge on DC Network Theorems to students.
- To familiarize students with the applications of theorems.

Syllabus:

Superposition, reciprocity, Thevenin's, Norton's, and Maximum power transfer theorems and simple problems. (All the above topics are only elementary treatment and simple problems).

Learning Outcomes:

Student will be able to

- Explain the importance of network theorems.
- Apply the correct theorem for solving complex networks.

DC NETWORK THEOREMS

Superposition theorem

Statement:

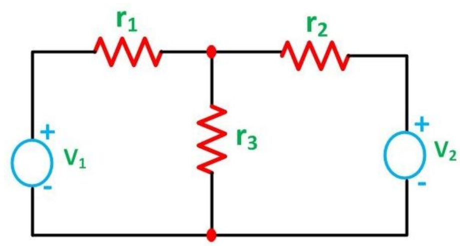
Superposition theorem states that, the response in any linear, bilateral network containing more than one sources is the sum of the responses produced by the sources each acting independently.

In other words, it can be stated as if a number of voltage or current sources are acting in a linear network, the resulting current in any branch is the algebraic sum of all the currents that would be produced in it, when each source acts alone, all the other independent sources are replaced by their internal resistances.

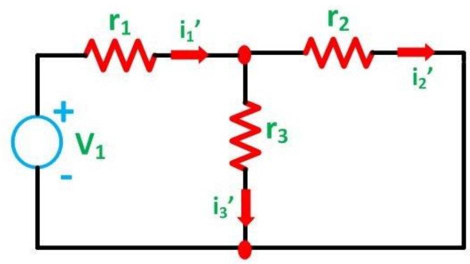
The superposition theorem is not applicable for the power, as power is directly proportional to the square of the current which is not a linear function.

Explanation of Superposition Theorem

Let us understand the superposition theorem with the help of an example. The circuit diagram shown below consists of a two voltage sources V_1 and V_2.



First, take the source V_1 alone and short circuit the V_2 source as shown in the circuit diagram below



Here, the value of current flowing in each branch, i.e. i_1', i_2' and i_3' is calculated by the following equations.

$$i_1' = \frac{V_1}{\frac{r_2 r_3}{r_2 + r_3} + r_1}$$

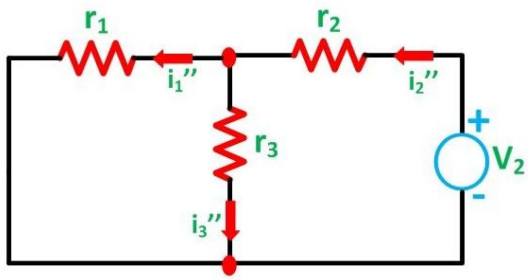
$$i_2' = i_1' \frac{r_3}{r_2 + r_3}$$

The difference between the above two equations gives the value of the current i3'

$$i_3' = i_1' - i_2'$$

Now, activating the voltage source V_2 and deactivating the voltage source

V_1 by short circuiting it, find the various currents, i.e. i_1'', i_2'', i_3'' flowing in the circuit diagram shown below



Here,

$$i_2'' = \frac{V_2}{\frac{r_1 r_3}{r_1 + r_3} + r_2} \quad \text{and} \quad i_1'' = i_2'' \frac{r_3}{r_1 + r_3}$$

And the value of the current i_3'' will be calculated by the equation shown below

$$i_3'' = i_2'' - i_1''$$

As per the superposition theorem the value of current i_1, i_2, i_3 is now calculated as

$$i_3 = i_3' + i_3''$$

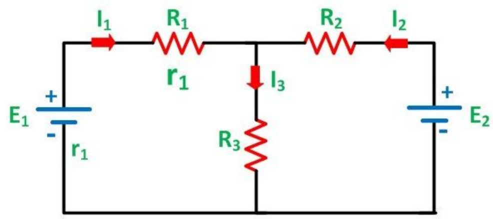
$$i_2 = i_2' - i_2''$$

$$i_1 = i_1' - i_1''$$

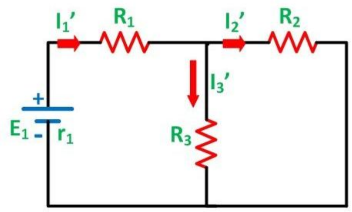
Direction of current should be taken care while finding the current in the various branches.

Steps for Solving network by Superposition Theorem

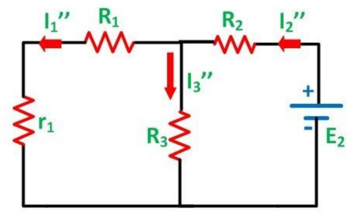
Considering the circuit diagram A, let us see the various steps to solve the superposition theorem



Circuit Diagram A



Circuit Diagram B



Circuit Diagram C

Step 1 – Take only one independent source of voltage or current and deactivate the other source.

Step 2 – In the circuit diagram B shown above, consider the source E_1 and replace the other source E_2 by its internal resistance. If its internal resistance is not given, then it is taken as zero and the source is short circuited.

Step 3 – If there is a voltage source than short circuit it and if there is a current source than just open circuit it.

Step 4 – Thus, by activating one source and deactivating the other source find the current in each branch of the network. Taking above example find the current I_1', I_2'and I_3'.

Step 5 – Now consider the other source E_2 and replace the source E_1 by its internal resistance r_1 as shown in the circuit diagram C.

Step 6 – Determine the current in various sections, I_1'', I_2'' and I_3''.

Step 7 – Now to determine the net branch current utilizing the superposition theorem, add the currents obtained from each individual source for each branch.

Step 8 – If the current obtained by each branch is in the same direction than add them and if it is in the opposite direction, subtract them to obtain the net current in each branch.

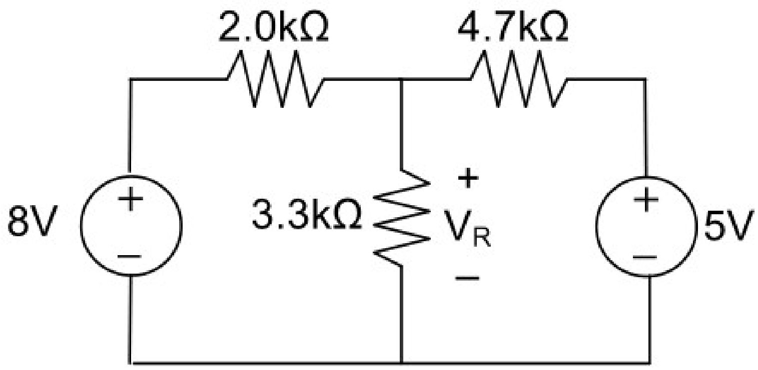
The actual flow of current in the circuit C will be given by the equations shown below

$$I_1 = I_1' - I_1''$$

$$I_2 = I_2' - I_2''$$

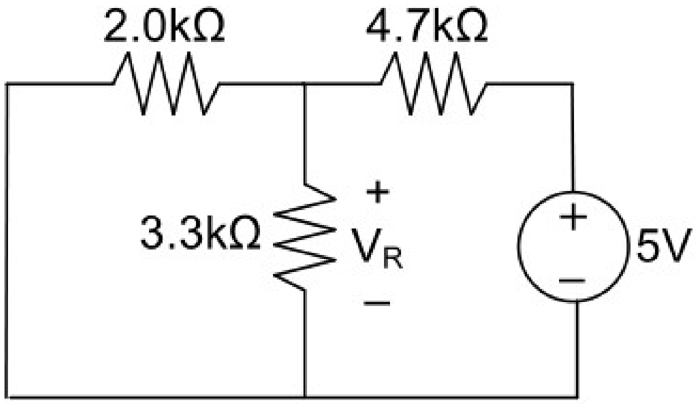
$$I_3 = I_3' - I_3''$$

Ex: 1. Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in figure below.



Solution:

Step 1: Remove the 8V power supply from the original circuit, such that the new circuit becomes as the following and then measure voltage across resistor.

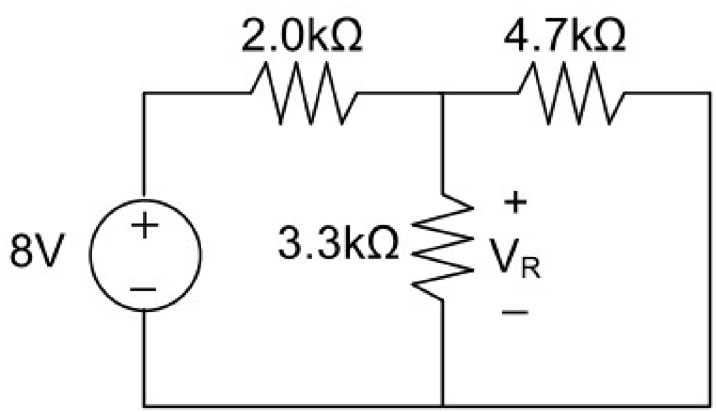


Here 3.3K and 2K are in parallel, therefore resultant resistance will be 1.245K.

Using voltage divider rule voltage across 1.245K will be

V1= [1.245/(1.245+4.7)]*5 = 1.047V

Step 2: Remove the 5V power supply from the original circuit such that the new circuit becomes as the following and then measure voltage across resistor.



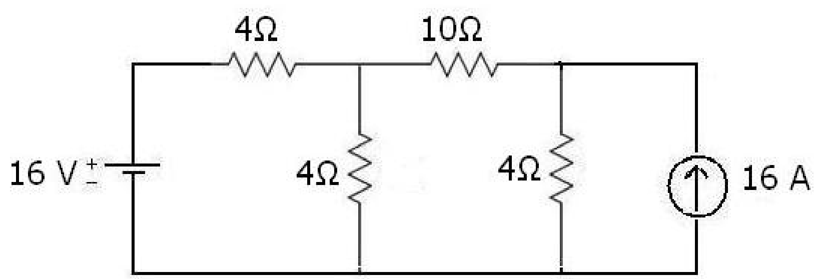
Here 3.3K and 4.7K are in parallel, therefore resultant resistance will be 1.938K.

Using voltage divider rule voltage across 1.938K will be

V2= [1.938/(1.938+2)]*8 = 3.9377V

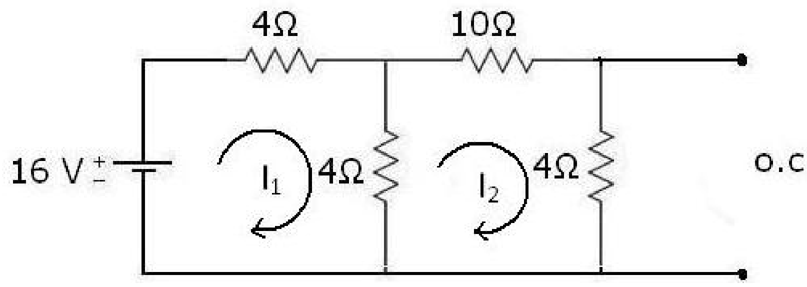
Therefore voltage drop across 3.3K resistor is V1+V2 = 1.047+3.9377=4.9847V.

Ex.2. Find the current through 10 Ω resistance in the given network by using superposition theorem?



For finding current through 10Ω resistance by using superposition theorem, we follows same step as we discussed in previous post.

Activating '16V' source at a time, other will be deactivated.



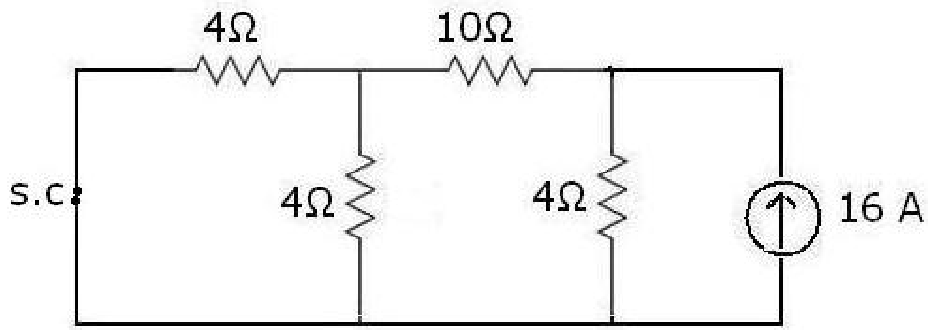
After deactivation of '16A' current source, two individual box are seem clearly in which we can easily applying mesh analysis for finding current through 10Ω resistance when '16V' voltage source is active. You can also use nodal analysis or ohm's law with current division rule.

So, after solving this circuit, we get

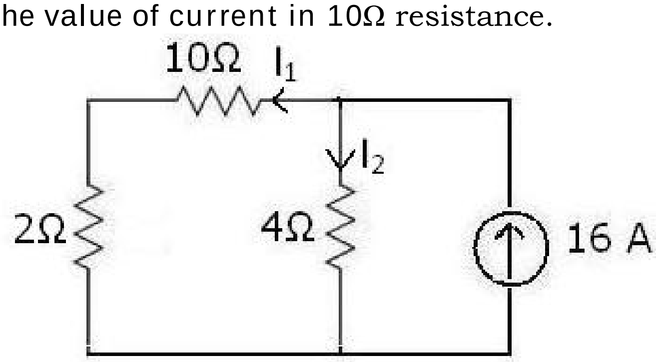
$I_1 =$ 2.25A & $I_2 =$ 0.5A

Then, $I_{10\Omega} = I_2 = 0.5A$

Activating '16A' source at a time, other will be deactivated.



After deactivation of '16V' voltage source, we can one step reduced circuit and applying current division rule. By applying current division rule, we can easily find the value of current in 10Ω resistance.



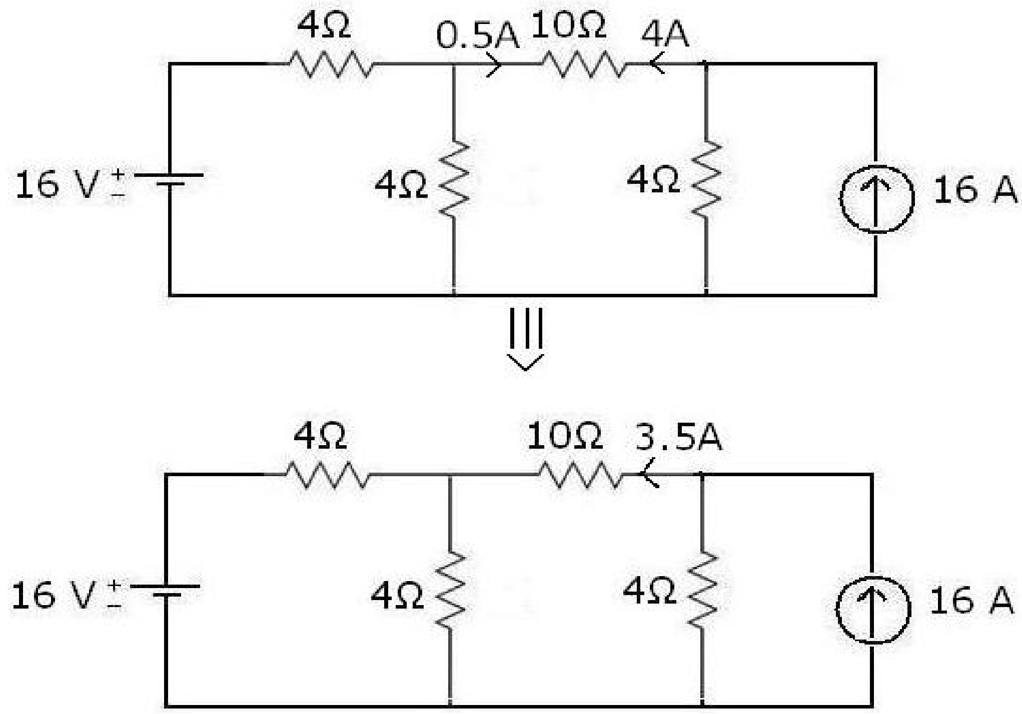
By current division rule,

$I_1 = 4A$

$I_2 = 12A$

So, $I_{10\Omega} = I_1 = 4A$

At last, the current through 10Ω resistance is 3.5A and greater current has their direction as shown below.



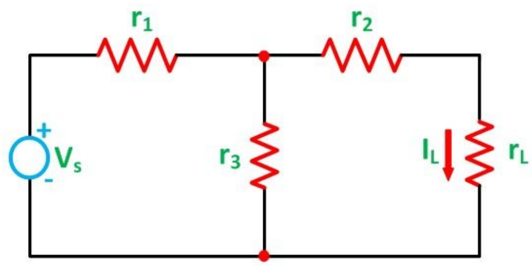
Thevenin's Theorem

Thevenin's Theorem states that – any linear, active, bilateral complicated network across its load terminals can be replaced by a single voltage source with one resistance in series.

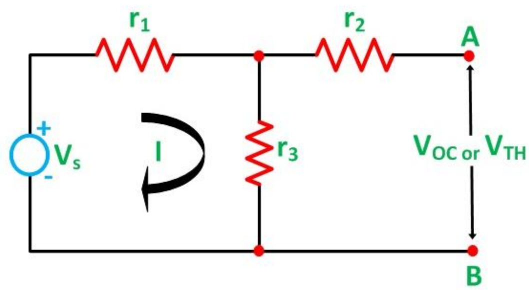
A more general statement of Thevenin's Theorem is that any linear active network consisting of independent or dependent voltage and current source and the network elements can be replaced by an equivalent circuit having a voltage source in series with a resistance, that voltage source being the open circuited voltage across the open circuited load terminals and the resistance being the internal resistance of the source**.**

Explanation of Thevenin's Theorem

The Thevenin's statement is explained with the help of a circuit shown below.

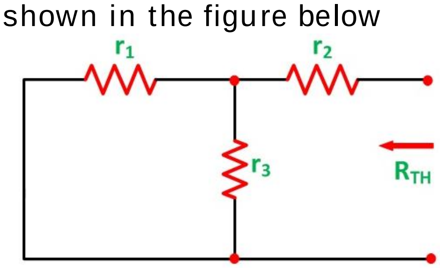


Let us consider a simple DC circuit as shown in the figure above, where we have to find the load current $\mathbf{I_L}$ by the Thevenin's theorem. In order to find the equivalent voltage source, $\mathbf{r_L}$ is removed from the circuit as shown in the figure below and $\mathbf{V_{oc}}$ or $\mathbf{V_{TH}}$ is calculated

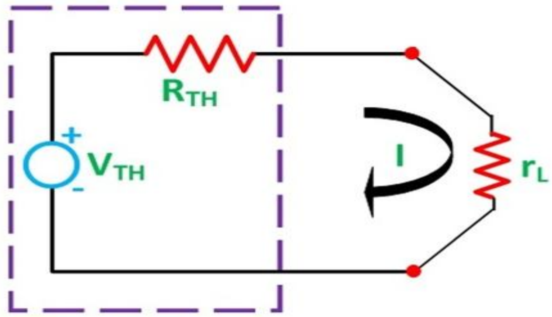


$$V_{OC} = I\, r_3 = \frac{V_S}{r_1 + r_3}\, r_3$$

Now, to find the internal resistance of the network (Thevenin's resistance or equivalent resistance) in series with the open circuit voltage $\mathbf{V_{OC}}$, also known as Thevenin's voltage $\mathbf{V_{TH}}$, the voltage source is removed or we can say it is deactivated by a short circuit (as the source does not have any internal resistance) as shown in the figure below



$$R_{TH} = r_2 + \frac{r_1 r_3}{r_1 + r_3}$$



Equivalent Circuit of Thevenin's Theorem

As per Thevenin's Statement, the load current is determined by the circuit shown above and the equivalent Thevenin's circuit is obtained.
The Load current $\mathbf{I_L}$ is given as

$$I_L = \frac{V_{TH}}{R_{TH} + r_L}$$

Where, $\mathbf{V_{TH}}$ is the Thevenin's equivalent voltage. It is an open circuit voltage across the terminal AB known as load terminal

$\mathbf{R_{TH}}$ is the Thevenin's equivalent resistance, as seen from the load terminals where all the sources are replaced by their internal impedance
$\mathbf{r_L}$ is the load resistance

Steps for Solving Thevenin's Theorem

Step 1 – First of all remove the load resistance $\mathbf{r_L}$ of the given circuit.

Step 2 – Replace all the impedance source by their internal resistance.

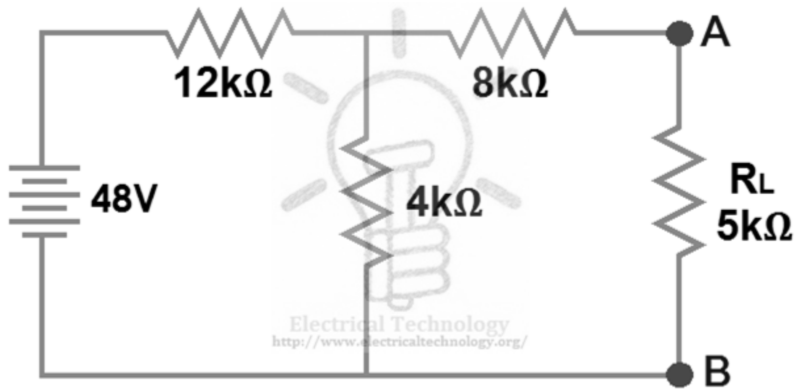
Step 3 – If sources are ideal then short circuit the voltage source and open the current source.

Step 4 – Now find the equivalent resistance at the load terminals know as Thevenin's Resistance (R_{TH}).

Step 5 – Draw the Thevenin's equivalent circuit by connecting the load resistance and after that determine the desired response.

This theorem is possibly the most extensively used networks theorem. It is applicable where it is desired to determine the current through or voltage across any one element in a network. The Thevenin's Theorem is an easy way to solve the complicated network.

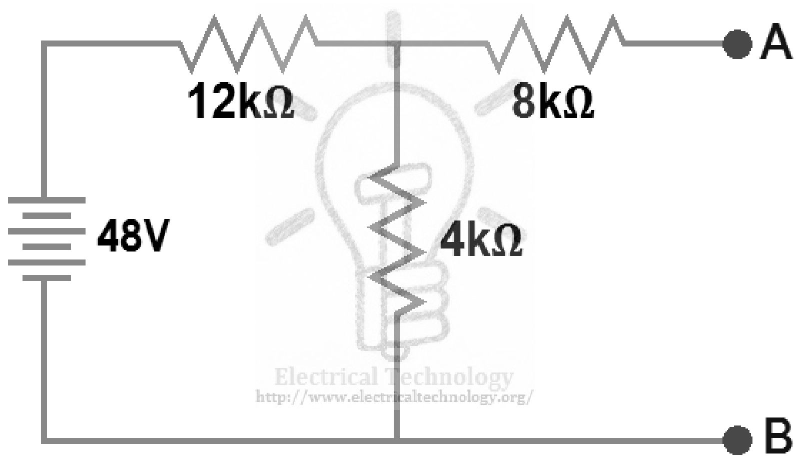
Ex1. Find V_{TH}, R_{TH} and the load current flowing through and load voltage across the load resistor by using Thevenin's Theorem.



Solution:-

Step 1.

Open the 5kΩ load resistor.

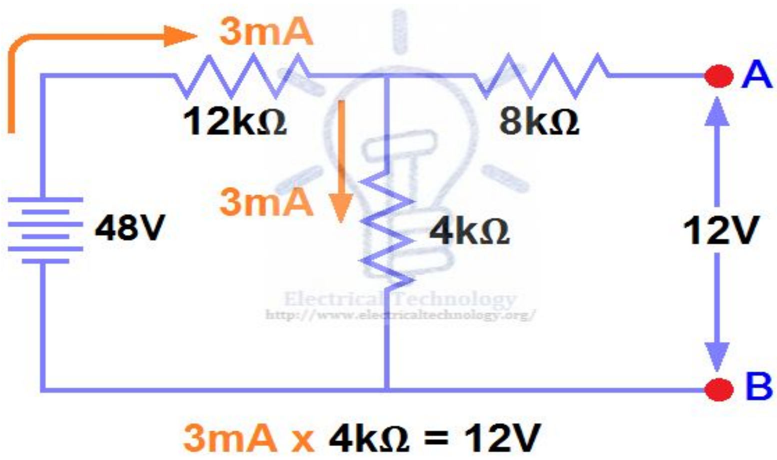


Step 2.

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V_{TH}). We have already removed the load resistor from figure, so the circuit became an open circuit. Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open.

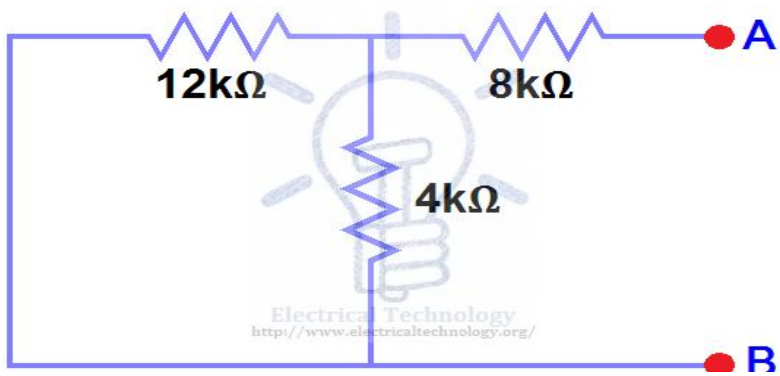
So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals. So,

V_{TH} = 12V



Step 3.

Open Current Sources and Short Voltage Sources



Step 4.

Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R_{TH})

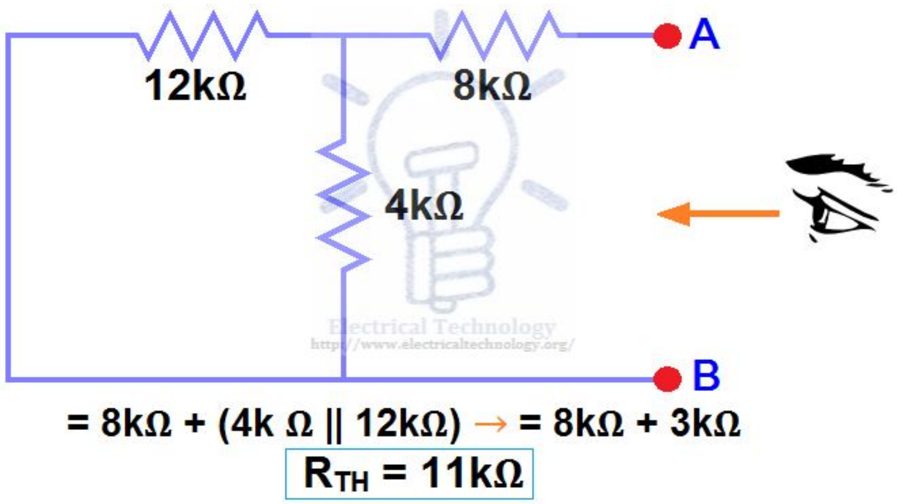
We have Reduced the 48V DC source to zero is equivalent to replace it with a short in step (3), as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R_{TH} = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

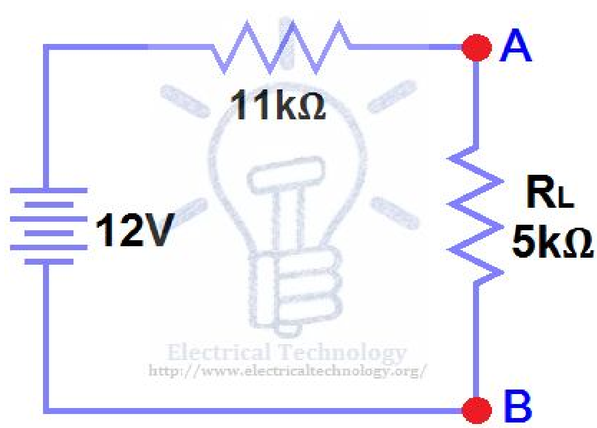
R_{TH} = 8kΩ + 3kΩ

R_{TH} = 11kΩ



Step 5.

Connect the R_{TH}in series with Voltage Source V_{TH} and re-connect the load resistor. This is shown in fig (6) i.e. Thevenin circuit with load resistor. This the Thevenin's equivalent circuit



Then calculate the current in the load resistor, the total resistance in the circuit is 16K ohms. So, i= 12V/16Kohm = 0.75mA.

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem can be stated as – A resistive load, being connected to a DC network, receives maximum power when the load resistance is equal to the internal resistance known as (Thevenin's equivalent resistance) of the source network as seen from the load terminals.

The Maximum Power Transfer theorem is used to find the load resistance for which there would be the maximum amount of power transfer from the source to the load.

Explanation of Maximum Power Transfer Theorem

A variable resistance R_L is connected to a DC source network as shown in the circuit diagram in figure A below and the figure B represents the Thevenin's voltage V_{TH} and Thevenin's resistance R_{TH} of the source network. The aim of the Maximum Power Transfer theorem is to determine the value of load resistance R_L, such that it receives maximum power from the DC source.

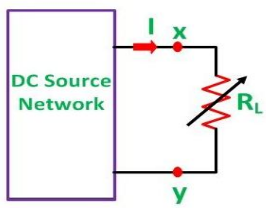


Figure A

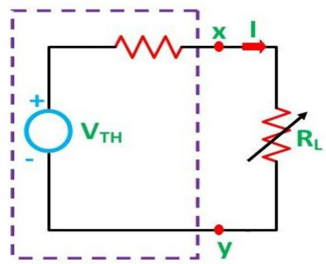


Figure B

Considering figure B the value of current will be calculated by the equation shown below

$$I = \frac{V_{TH}}{R_{TH} + R_L}$$

While the power delivered to the resistive load is given by the equation

$$P_L = I^2 R_L$$

Putting the value of I from the equation (1) in the equation (2) we will get

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L}\right)^2 x\, R_L$$

P_L can be maximized by varying R_L and hence, maximum power can be delivered when

$$(dP_L / dR_L) = 0$$

However,

$$\frac{dP_L}{dR_L} = \frac{1}{[(R_{TH} + R_L)^2]^2}\left[(R_{TH} + R_L)^2 \frac{d}{dR_L}(V_{TH}^2 R_L) - V_{TH}^2 R_L \frac{d}{dR_L}(R_{TH} + R_L)^2\right]$$

$$\frac{dP_L}{dR_L} = \frac{1}{(R_{TH} + R_L)^4}[(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L \text{ x } 2(R_{TH} + R_L)]$$

$$\frac{dP_L}{dR_L} = \frac{V_{TH}^2 (R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} = \frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2}$$

But as we know, $(dP_L / dR_L) = 0$

Therefore,

$$\frac{V_{TH}^2 (R_{TH} - R_L)}{(R_{TH} + R_L)^2} = 0$$

Which gives

$$(R_{TH} - R_L) = 0 \quad \text{or} \quad R_{TH} = R_L$$

Hence, it is proved that power transfer from a DC source network to a resistive network is maximum when the internal resistance of the DC source network is equal to the load resistance.

Again, with $R_{TH} = R_L$, the system being perfectly matched to the load and the source, thus, the power transfer becomes maximum, and this amount of power Pmax can be obtained by the equation shown below

$$P_{max} = \frac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2} = \frac{V_{TH}^2}{4R_{TH}}$$

Above equation gives the power which is consumed by the load. The power transfer by the source will also be same as the power consumed by the load, i.e. above equation, as the load power and the source power being the same.

Thus, the total power supplied is given by the equation

$$P = 2\,\frac{V_{TH}^2}{4R_{TH}} = \frac{V_{TH}^2}{2R_{TH}}$$

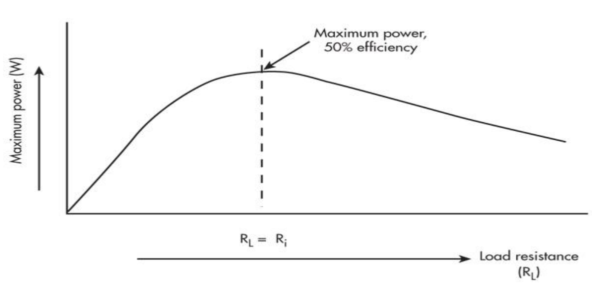
During Maximum Power Transfer the efficiency η becomes

$$\eta = \left(\frac{P_{max}}{P}\right) x\,100 = 50\%$$

The concept of Maximum Power Transfer theorem is that by making the source resistance equal to the load resistance, which has wide application in communication circuits where the magnitude of power transfer is sufficiently small. To achieve maximum power transfer, the source and the load resistance are matched and with this, efficiency becomes 50% with the flow of maximum power from the source to the load.

In Electrical Power Transmission system, the load resistance being sufficiently greater than the source resistance, it is difficult to achieve the condition of maximum power transfer.

In power system emphasis is given to keep the voltage drops and the line losses to a minimum value and hence the operation of the power system, operating with bulk power transmission capability, becomes uneconomical if it is operating with only 50% efficiency just for achieving maximum power transfer. Hence, in the electrical power transmission system, the criterion of maximum power transfer is very rarely used.



Steps for Solving Network Using Maximum Power Transfer Theorem

Following steps are used to solve the problem by Maximum Power Transfer theorem

Step 1 – Remove the load resistance of the circuit.

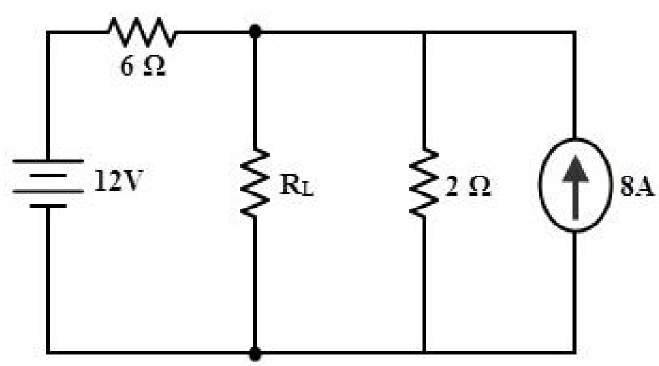
Step 2 – Find the Thevenin's resistance (R_{TH}) of the source network looking through the open circuited load terminals.

Step 3 – As per the maximum power transfer theorem, this R_{TH} is the load resistance of the network, i.e., $R_L = R_{TH}$ that allows maximum power transfer.

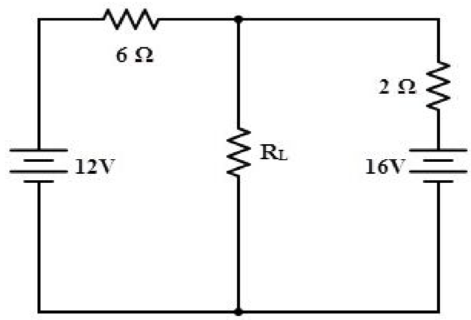
Step 4 – Maximum Power Transfer is calculated by the equation shown below

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}$$

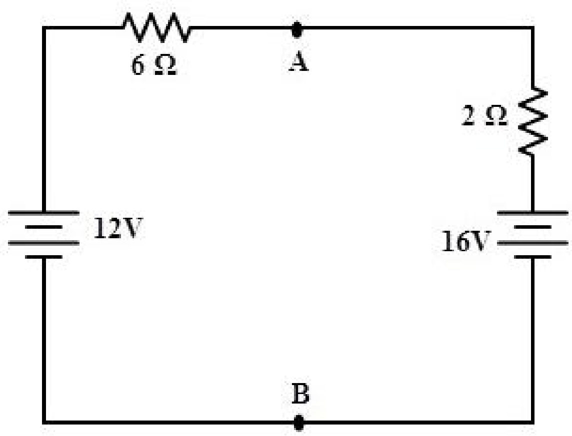
Ex: Consider the below circuit for which we are going to determine the value of load resistance, R_L for which maximum power will transfer from source to load.



Now, the given circuit can be further simplified by converting the current source into equivalent voltage source as follows.



We need to find the Thevenin's equivalent voltage Vth and Thevenin's equivalent resistance Rth across the load terminals in order to get the condition for maximum power transfer. By disconnecting the load resistance, the open-circuit voltage across the load terminals can be calculated as;



By applying Kirchhoff's voltage law, we get

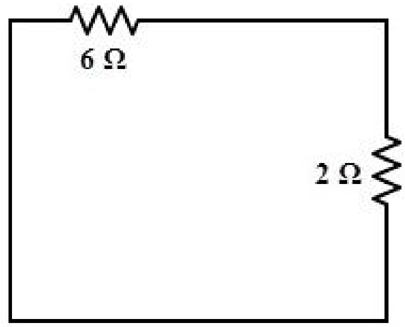
$$12 - 6I - 2I - 16 = 0$$

$$-8I = 4$$

$$I = -0.5\ A$$

The open-circuit voltage across the terminals A and B, $V_{AB} = 16 - 2 \times 0.5 = 15$ V

Thevenin's equivalent resistance across the terminals A and B is obtained by short-circuiting the voltage sources as shown in the figure.



Req = (6 × 2) / (6 + 2)

= 1.5 Ω

So the maximum power will transferred to the load when R_L = 1.5 ohm.

Current through the circuit, I = 15 / (1.5 + 1.5)

= 5 A

Therefore, the maximum power = $5^2 \times 1.5 = 37.5$ W

Reciprocity Theorem

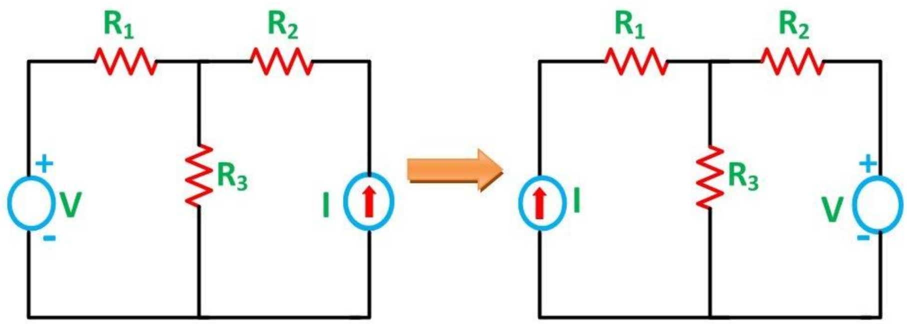
Reciprocity Theorem states that – In any branch of a network or circuit, the current due to a single source of voltage (V) in the network is equal to the current through that branch in which the source was originally placed when the source is again put in the branch in which the current was originally obtained.

In simple words, we can state the reciprocity theorem as when the places of voltage and current source in any network is interchanged the amount or magnitude of current and voltage flowing in the circuit remains the same

Explanation of Reciprocity Theorem

The location of the voltage source and the current source may be interchanged without a change in current. However, the polarity of the voltage source should be identical with the direction of the branch current in each position.

The Reciprocity Theorem is explained with the help of the circuit diagram shown below



The various resistances R_1, R_2, R_3 is connected in the circuit diagram above with a voltage source (V) and a current source (I). It is clear from the figure above that the voltage source and current sources are interchanged for solving the network with the help of Reciprocity Theorem.

The limitation of this theorem is that it is applicable only to single source networks and not in the multi-source network. The network where reciprocity theorem is applied should be linear and consist of resistors, inductors, capacitors and coupled circuits. The circuit should not have any time-varying elements.

Steps for Solving a Network Utilizing Reciprocity Theorem

Step 1 – Firstly, select the branches between which reciprocity has to be established.

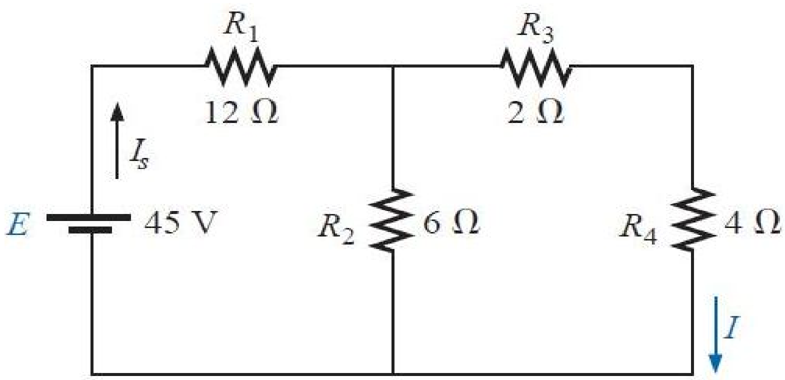
Step 2 – The current in the branch is obtained using any conventional network analysis method.

Step 3 – The voltage source is interchanged between the branch which is selected.

Step 4 – The current in the branch where the voltage source existed earlier is calculated.

Step 5 – Now, it is seen that the current obtained in the previous connection, i.e., in step 2 and the current which is calculated when the source are interchanged i.e., in step 4 are identical to each other.

Ex: Verify the Reciprocity Theorem for the following circuit.



Req= ((2+4) | | 6)+12 =15 ohm

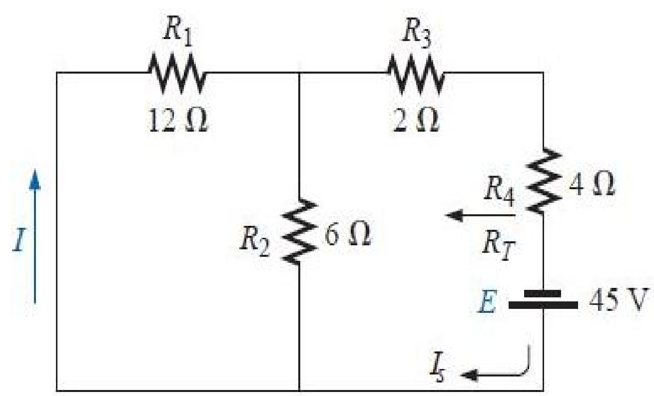
Is= 45/15= 3A

By using Current Division rule,

I= (3*6)/12 = 1.5A

Since I is pointed downwards in the given problem, so the positive terminal faced downwards when we interchange.

Since E positive terminal points upwards, the current also points upwards when we interchange.



Rt= ((12||6)+6) = 10ohms

Is= 45/10= 4.5A

I= (4.5*6)/18 = 1.5A

Hence in both the cases, I is equal. Hence Reciprocity theorem is verified.

UNIT II
Assignment Cum Tutorial

Section - A

Objective Questions

1) Super position theorem can be applied only to circuits having [ ]

a) Resistive elements b) passive elements

c) Non linear elements d) linear bilateral elements

2) In applying superposition theorem to determine branch currents and voltages, [ ]

a) All current & voltage sources are removed

b) Only the current sources are removed

c) Only one source (Current or Voltage) is included at a time

d) Only the voltage sources are removed

3) Thevenin's theorem can be applied to networks containing [ ]

a) Passive Elements only b) Active elements only

c) Linear elements only d) All of these

4) Which of the following Theorem helps in simplifying complications when the load across the circuits is varying? [ ]

a) Superposition b) Norton's

c) Thevenin's d) Maximum Power Transfer

5) Superposition theorem requires as many circuits to be solved as there are [ ]

a) Sources b)Nodes

c) Sources and Nodes d) Sources, Nodes and Meshes

6) In deriving the equivalent resistance at any pair of terminals of a network with the help of thevenin's theorem [ ]

a) All independent voltage circuits are open circuited

b) All independent current sources are short circuited

c) The internal resistance of all independent sources is neglected

d) None of these

7) For the maximum power transfer to the load [ ]

a) Load resistance should be twice the internal resistance of the voltage source

b) Load resistance should be equal the internal resistance of the voltage source

c) Load resistance should be half the internal resistance of the voltage source

d) None of these

8) Superposition theorem is not valid for [ ]

a) Voltage responses b) Current responses

c) Power responses d) All of these

9) In balanced bridge, if the positions of detector and source are interchanged, the bridge will still remain balanced. This can be explained from which theorem [ ]

a) Reciprocity theorem b) Thevenin's theorem

c) Superposition theorem d) Maximum power transfer theorem

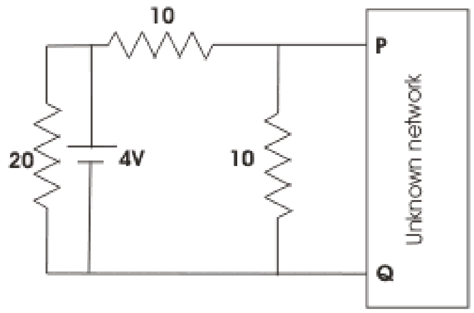
10) When a source is delivering maximum power to a load, the efficiency of the circuit [ ]

a) Always 50% b) Depends on the circuit parameters

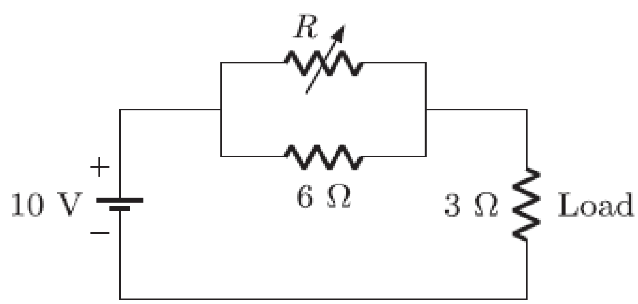
c) Always 75% d) None

11) In the given figure, the Thevenin's equivalent voltage and impedance as seen from the terminals P - Q is given by [ ]

a) 6 A b)4A c) 5 A d) 10 A



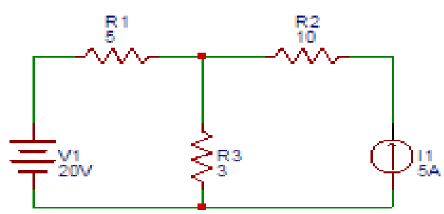
12) In the circuit given below, the value of *R* required for the transfer of maximum power to the load having a resistance of 3 Ω is [ ]



(a) Zero (b) 3 Ω (c) 6 Ω (d) infinity

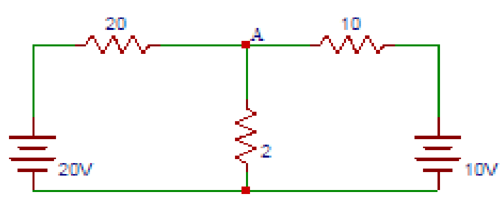
13) In the circuit shown, find the current through 4Ω resistor using Superposition theorem.

Superposition Theorem [ ]



a) 4 b) 5 c) 6 d) 7

14) Find the voltage across 2Ω resistor due to 20V source in the circuit shown below [ ]



a) 1 b) 1.5 c) 2 d) 2.5

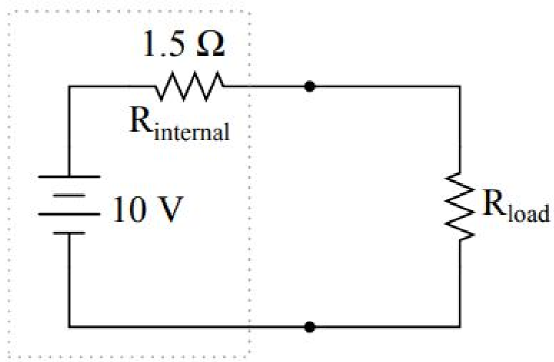
15) Find the voltage across 2Ω resistor due to 20V source in the circuit shown above. [ ]

a) 0.5 b) 0 c) 1 d) 1.5

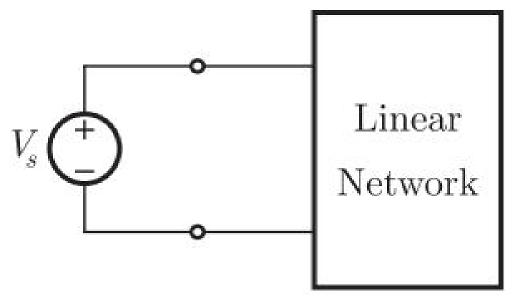
16) Find the voltage across 2Ω resistor in the circuit shown above using Superposition theorem. [ ]

a) 1 b) 2 c) 3 d) 4

17) The value of R_{load} for maximum power transfer [ ]

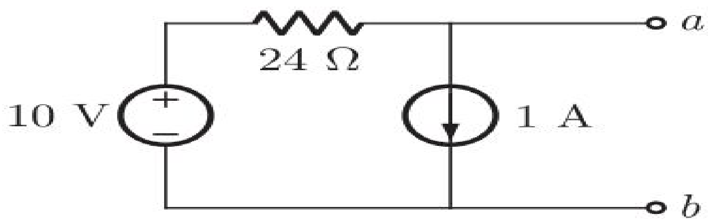


18) The linear network in the figure contains resistors and dependent sources only. When Vs = 10 V, the power supplied by the voltage source is 40 W. What will be the power supplied by the source if Vs = 5 V ? [ ]



(a) 20 W (b) 10 W (c) 40 W (d) can not be determined

19) For the circuit shown in the figure the Thevenin voltage and resistance seen from the terminal a-b are respectively [ ]



(A) 34 V, 0 Ω

(B) 20 V, 24 Ω

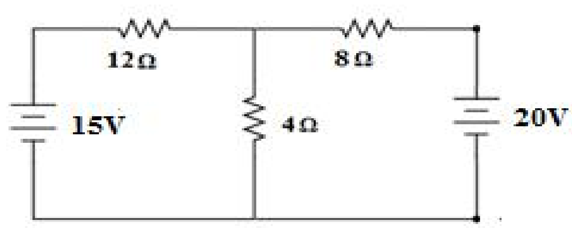
(C) 14 V, 0 Ω

(D) -14 V, 24 Ω

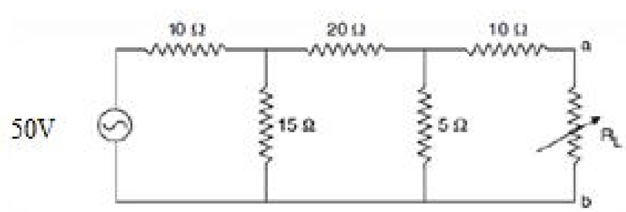
Section - B

Descriptive Questions

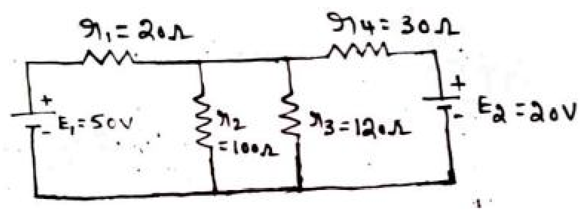
1. State and explain thevenin's theorem with its equivalent circuit
2. State and explain Superposition theorem.
3. State and explain Maximum power transfer theorem.
4. Prove that maximum power tranfered from load to source when load resistance is equal to the source resistance.
5. State and prove Reciprocity Theorem?
6. Calculate current flowing through 4Ω Resistor for the given circuit using super position theorem



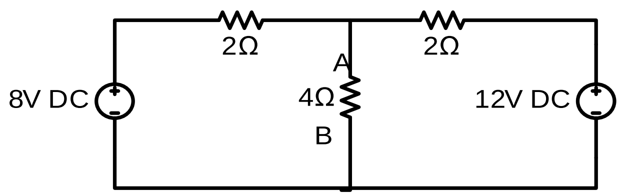
7. Determine the value of R_L in the network shown fig E2.17 for maximum power transfer and calculate the value of power



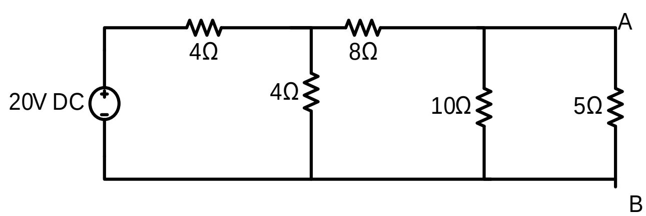
8. Using Thevenin's theorem, find the current through r2 in the circuit shown in below figure.



9. Find Thevenin's equivalent of the network as shown in the figure at the terminals A-B Determine the current through the load resistor of 4Ω connected across the terminals A,B ?



10. Apply Thevenin's theorem to calculate the current flowing through 5Ω resistor across A-B in the below figure?



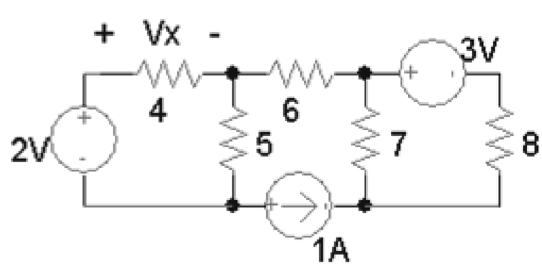
11.The equivalent circuit of a network represented by Thevenin's equivalent with V_{th} =12V, R_{th}=8V is connected to a load resistance R_L. If the conditions for maximum power transfer exist, determine

a) The value of R_L

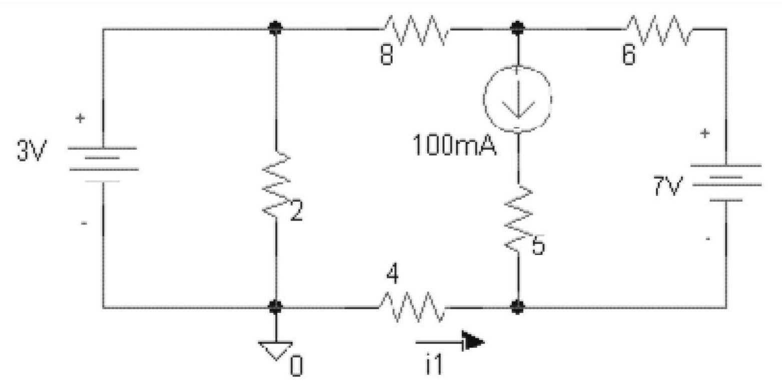
b) Power developed in R_L

c) Efficiency of the circuit i.e. Ratio of power absorbed by the load to power supplied by the source.

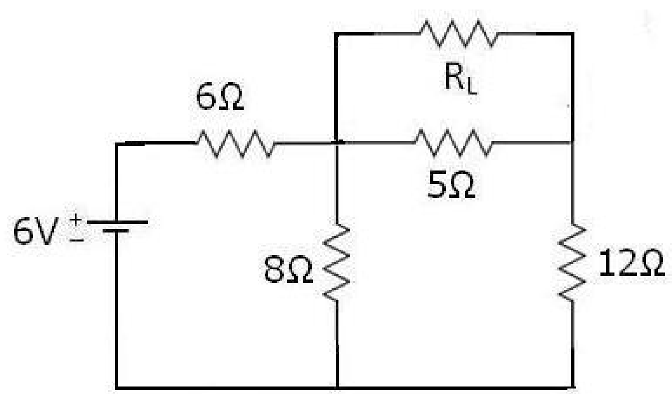
12. Find the voltage Vx using superposition.



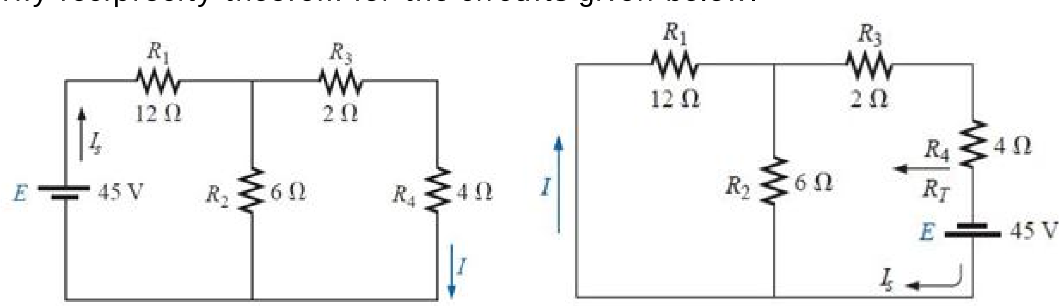
13. Solve for the current I1 (through the 4 ohm resistor) in the circuit shown above using superposition.



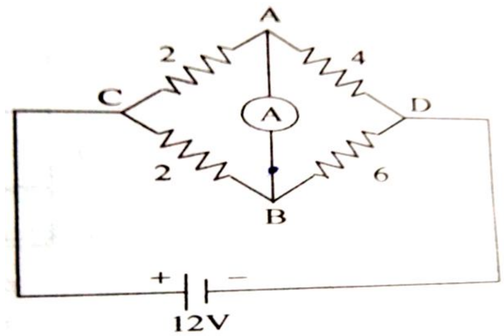
14. Find the value of R_L for the given network below that the power is maximum? And also find the Max Power through load-resistance R_L by using maximum power transfer theorem?



15. Verify reciprocity theorem for the circuits given below.

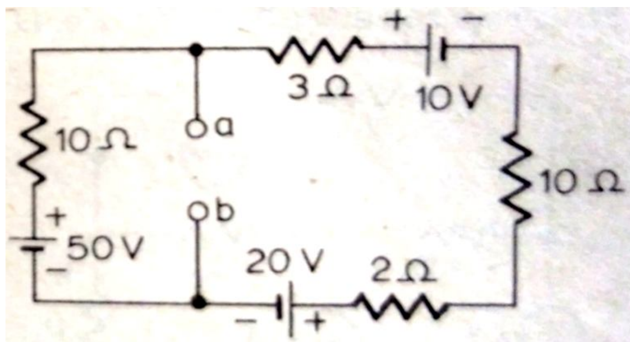


16. Determine the current through the ammeter of 2Ω , connected in the unbalanced Wheatstone bridge shown in the figure by applying thevenin's theorem

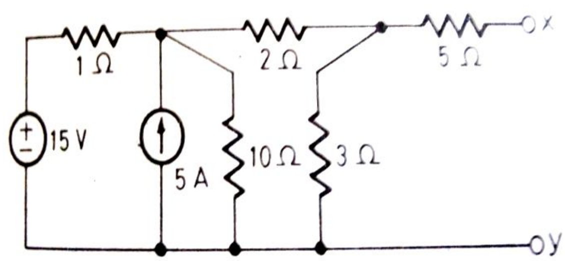


Wheatstone Bridge

17. Using superposition theorem, find the current through the link that is to be connected between terminals a-b. Assume the link resistance to be zero.



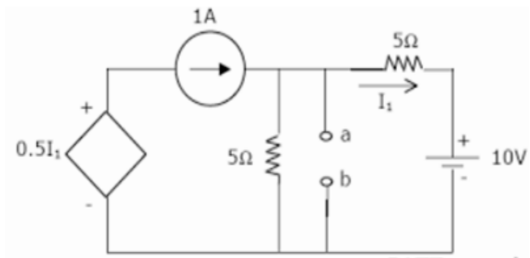
18. What resistance should be connected across x-y in the circuit shown in the fig. Such that maximum power is developed across this load resistance? What is the amount of maximum power?



SECTION – C

Gate and IES Questions

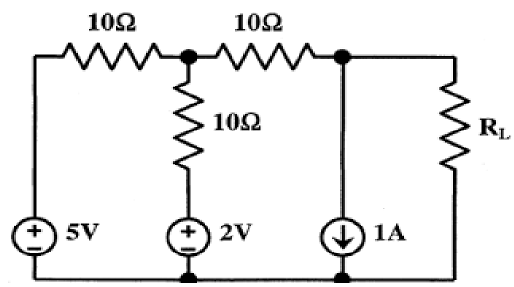
1. For the circuit shown in the fig, Thevenins voltage and thevenins equivalent resistance at the terminals a-b is **GATE 2005** **[ ]**



a) 5V and 2Ω b)7.5 Vand 2.5 Ω

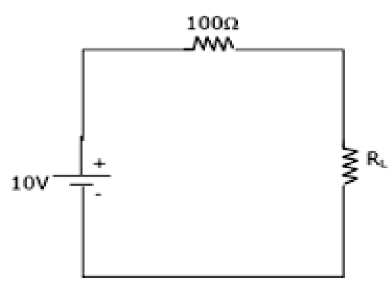
c) 4V and 2 Ω d) 3V and 2.5 Ω

2. In the circuit shown below, find the value of R_L such that power transferred to R_L is maximum. **GATE 2011** [ ]



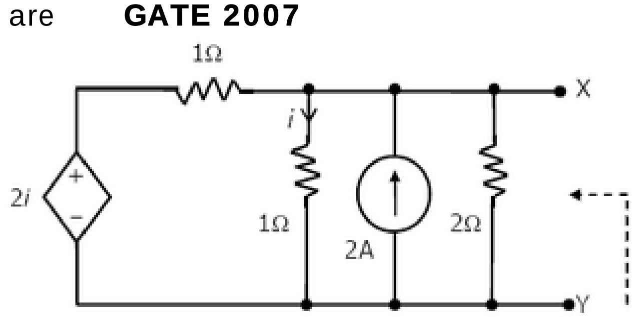
a) 5 ohms b) 10 ohms c) 15 ohms d) 20 ohms

3. The maximum power that can be transferred to the load resistor RL from the voltage source is **GATE 2005** [ ]



a) 1W b) 10W c) 0.25W d) 0.5W

4. For the circuit shown in the figure, the Thevenins Voltage and resistance looking into X-Y are **GATE 2007** [ ]



a) 4/3V, 3 ohm b) 4V, 2/3 ohm c) 4/3 V, 2/3 ohm d) 4V, 2 ohm

UNIT – III

D.C. Machines

Objectives:

1. To familiarize the students with the constructional details and working principle of DC machines.
2. To familiarize the students with characteristics of DC machines.

Syllabus:

Construction & Principle of operation of DC Generator – emf equation – types of DC Generators. (All the above topics are only elementary treatment and simple problems).

Learning Outcomes:

After the completion of this unit, students will be to

1. Explain the function of various parts of a dc machine.
2. Describe the working of a dc machine for generating action.
3. Determine the e.m.f of a dc machine.

D.C GENERATORS

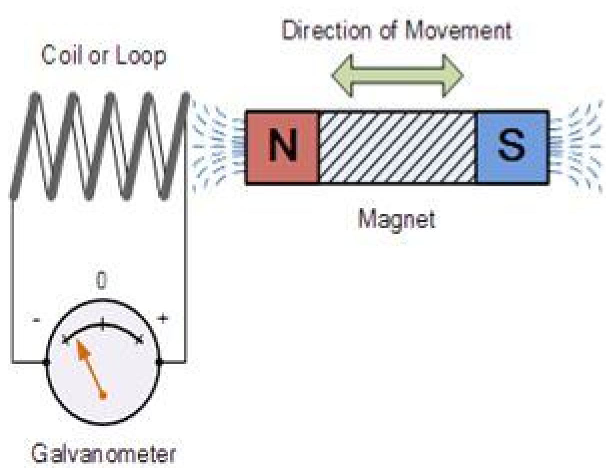
Generator principle

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). Induced e.m.f is produced in it according to Faraday's law of electromagnetic induction. This e.m.f causes a current to flow if the conductor circuit is closed.

Faraday’s Laws of Electromagnetic Induction:

In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's law of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors . This law shows the relationship between electric circuit and magnetic field. Faraday performs an experiment with a magnet and coil. During this experiment, he found how emf is induced in the coil when flux linked with it changes. He has also done experiments in electro-chemistry and electrolysis.

Faraday's Experiment



In this experiment, Faraday takes a magnet and a coil and connects a galvanometer across the coil. At starting, the magnet is at rest, so there is no deflection in the galvanometer i.e. needle of galvanometer is at the center or zero position. When the magnet is moved towards the coil, the needle of galvanometer deflects in one direction. When the magnet is held stationary at that position, the needle of galvanometer returns back to zero position. Now when the magnet is moved away from the coil, there is some deflection in the needle but in opposite direction and again when the magnet becomes stationary, at that point with respect to coil, the needle of the galvanometer returns back to the zero position. Similarly, if magnet is held stationary and the coil is moved away and towards the magnet, the galvanometer shows deflection in similar manner. It is also seen that, the faster the change in the magnetic field, the greater will be the induced emf or voltage in the coil.

CONCLUSION: From this experiment, Faraday concluded that whenever there is relative motion between conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage across a coil.

Faraday's First Law

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current. Method to change magnetic field:

1. By moving a magnet towards or away from the coil
2. By moving the coil into or out of the magnetic field.
3. By changing the area of a coil placed in the magnetic field
4. By rotating the coil relative to the magnet.

Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Consider a magnet approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time, $T_1 = N\Phi_1$ Wb

Flux linkage with the coil at time, $T_2 = N\Phi_2$ wb

Change in flux linkage = $N(\Phi_2 - \Phi_1)$

Let this change in flux linkage be, $\Phi = \Phi_2 - \Phi_1$

So, the Change in flux linkage = $N\Phi$

Now the rate of change of flux linkage = $N\Phi / t$

Take derivative on right hand side we will get

The rate of change of flux linkage = $Nd\Phi/dt$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = N\frac{d\emptyset}{dt}$$

Considering Lenz's Law.

$$E = -N\frac{d\emptyset}{dt}$$

Where flux Φ = B.A in Wb

B = magnetic field strength

A = area of the coil

Lenz's Law

Lenz's law states that when an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such, that it produces an current that's magnetic field opposes the change which produces it.

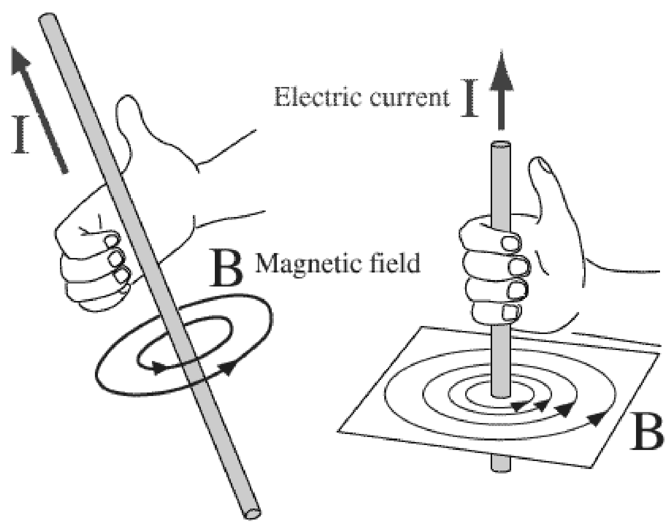
The negative sign used in Faraday's law of electromagnetic induction, indicates that the induced emf (E) and the change in magnetic flux ($d\Phi_B$) have opposite signs.

Where E = Induced emf

$d\Phi_B$ = change in magnetic flux

N = No of turns in coil

NOTE : For finding the directions of magnetic field or current, use right hand thumb rule i.e. if the fingers of the right hand are placed around the wire so that the thumb points in the direction of current flow, then the curling of fingers will show the direction of the magnetic field produced by the wire.

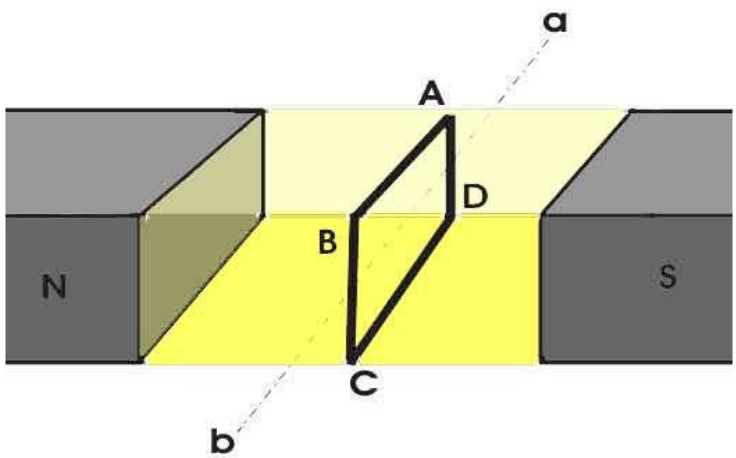


Principle of DC Generator

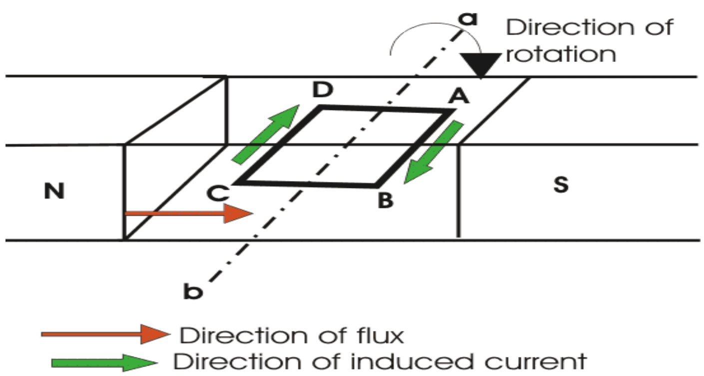
A DC generator produces direct power. These generators produce electrical power, based on fundamental principle of Faraday's law of electromagnetic induction. According to this law, when a conductor moves in a magnetic field it cuts magnetic lines force, due to which an emf is induced in the conductor. The magnitude of this induced emf depends upon the rate of change of flux (magnetic line force) linkage with the conductor. This emf will cause a current to flow if the conductor circuit is closed.

Hence the most basic two essential parts of a generator are a) a magnetic field and b) conductors which move inside that magnetic field.

Now we will go through working **principle of dc generator**.

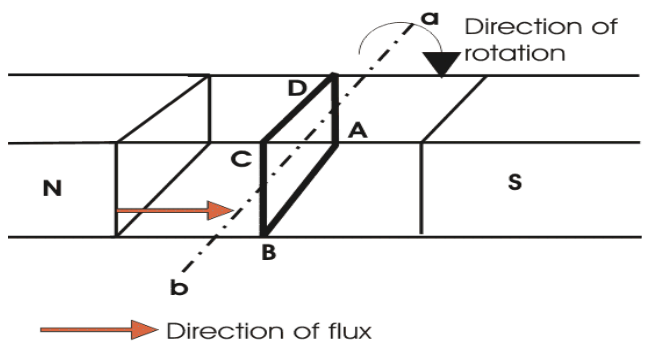


In the figure above, a single loop of conductor of rectangular shape is placed between two opposite poles of magnet. Let's us consider, the rectangular loop of conductor is ABCD which rotates inside the magnetic field about its own axis ab. When the loop rotates from its vertical position to its horizontal position, it cuts the flux lines of the field. As during this movement two sides, i.e. AB and CD of the loop cut the flux lines there will be an emf induced in these both of the sides (AB & BC) of the loop.



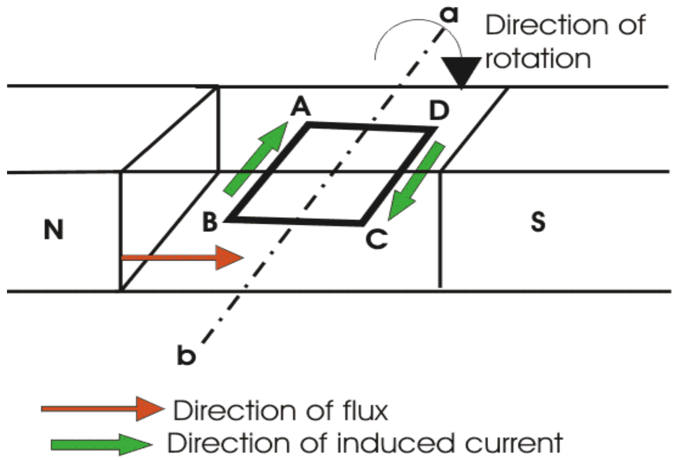
As the loop is closed there will be a current circulating through the loop. The direction of the current can be determined by Flemming's right hand Rule. This rule says that is you stretch thumb, index finger and middle finger of your right hand perpendicular to each other, then thumbs indicates the direction of motion of the conductor, index finger indicates the direction of magnetic field i.e. N - pole to S - pole, and middle finger indicates the direction of flow of current through the conductor.

Now if we apply this right hand rule, we will see at this horizontal position of the loop, current will flow from point A to B and on the other side of the loop current will flow from point C to D.



Now if we allow the loop to move further, it will come again to its vertical position, but now upper side of the loop will be CD and lower side will be AB (just opposite of the previous vertical position). At this position the tangential motion of the sides of the loop is parallel to the flux lines of the field. Hence there will be no question of flux cutting and consequently there will be no current in the loop.

If the loop rotates further, it comes to again in horizontal position. But now, said AB side of the loop comes in front of N pole and CD comes in front of S pole, i.e. just opposite to the previous horizontal position as shown in the figure beside.



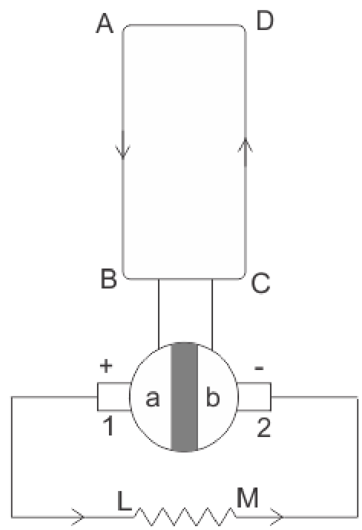
Here the tangential motion of the side of the loop is perpendicular to the flux lines, hence rate of flux cutting is maximum here and according to Flemming's right hand rule, at this position current flows from B to A and on other side from D to C.

Now if the loop is continued to rotate about its axis, every time the side AB comes in front of S pole, the current flows from A to B and when it comes in front of N pole, the current flows from B to A. Similarly, every time the side CD comes in front of S pole the current flows from C to D and when it comes in front of N pole the current flows from D to C.

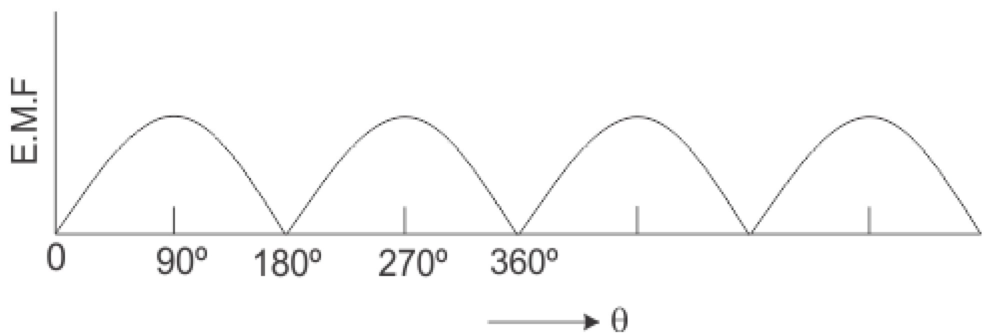
If we observe this phenomena in different way, it can be concluded, that each side of the loop comes in front of N pole, the current will flow through that side in same direction i.e. downward to the reference plane and similarly each side of the loop comes in front of S pole, current through it flows in same direction i.e. upwards from reference plane. From this, we will come to the topic of **principle of dc generator**.

Now the loop is opened and connects it with a split ring as shown in the figure below. Split rings are made out of a conducting cylinder which cuts into two halves or segments insulated from each other. The external load terminals are connected with two carbon brushes which are rest on these split slip ring segments.

Working Principle of DC Generator



It is seen that in the first half of the revolution current flows always along ABLMCD i.e. brush no 1 in contact with segment a. In the next half revolution, in the figure the direction of the induced current in the coil is reversed. But at the same time the position of the segments a and b are also reversed which results that brush no 1 comes in touch with that segment b. Hence, the current in the load resistance again flows from L to M. The waveform of the current through the load circuit is as shown in the figure. This current is unidirectional.



This is basic working principle of DC generator, explained by single loop generator model.

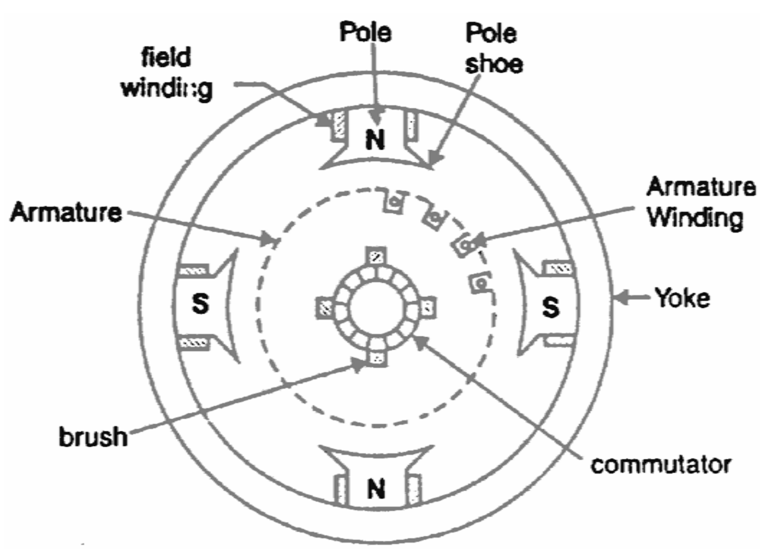
The position of the brushes of DC generator is so arranged that the changeover of the segments a and b from one brush to other takes place when the plane of rotating coil is at right angle to the plane of the lines of force. It is so become in that position, the induced emf in the coil is zero.

Construction of DC Machine (for both Generator and Motor)

A DC motor like we all know is a device that deals in the conversion of electrical energy to mechanical energy and this is essentially brought about by two major parts required for the **construction of dc motor**, namely.

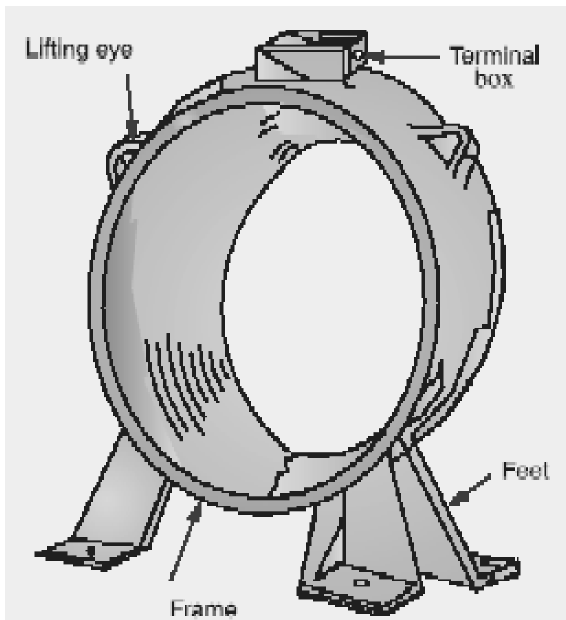
1) Stator – The static part that houses the field windings and receives the supply and,
2) Rotor – The rotating part that brings about the mechanical rotations.
3) Yoke of dc motor.
4) Poles of dc motor.
5) Field winding of dc motor.
6) Armature winding of dc motor.
7) Commutator of dc motor.
8) Brushes

All these parts put together configures the total **construction of a dc motor**. Now let's do a detailed discussion about all the essential parts of dc motor.



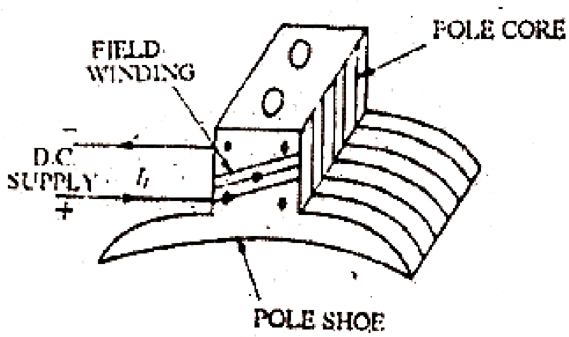
Yoke of DC Machine

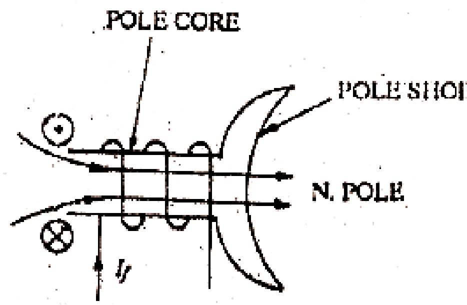
The magnetic frame or the **yoke of dc motor** made up of cast iron or steel and forms an integral part of the stator or the static part of the motor. Its main function is to form a protective covering over the inner sophisticated parts of the motor and provide support to the armature. It also supports the field system by housing the magnetic poles and field winding of the dc motor.



Poles of DC Machine

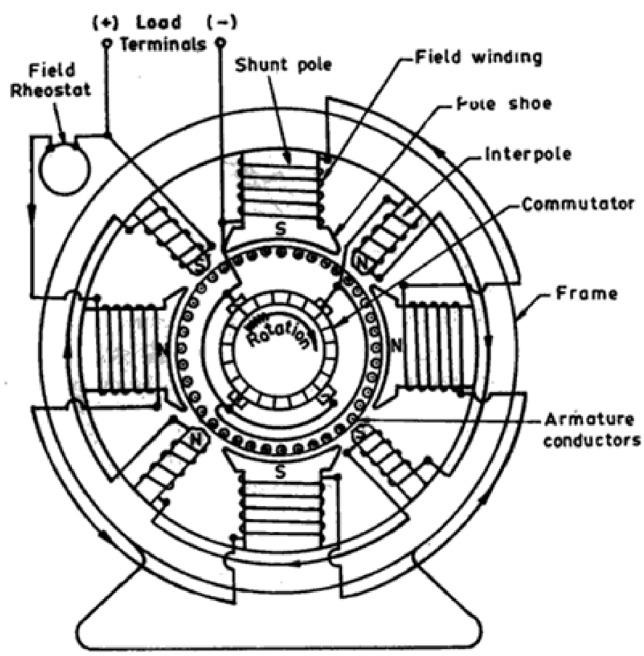
The magnetic **poles of DC motor** are structures fitted onto the inner wall of the yoke with screws. The construction of magnetic poles basically comprises of two parts namely, the pole core and the pole shoe stacked together under hydraulic pressure and then attached to the yoke. These two structures are assigned for different purposes, the pole core is of small cross sectional area and its function is to just hold the pole shoe over the yoke, whereas the pole shoe having a relatively larger cross-sectional area spreads the flux produced over the air gap between the stator and rotor to reduce the loss due to reluctance. The pole shoe also carries slots for the field windings that produce the field flux.

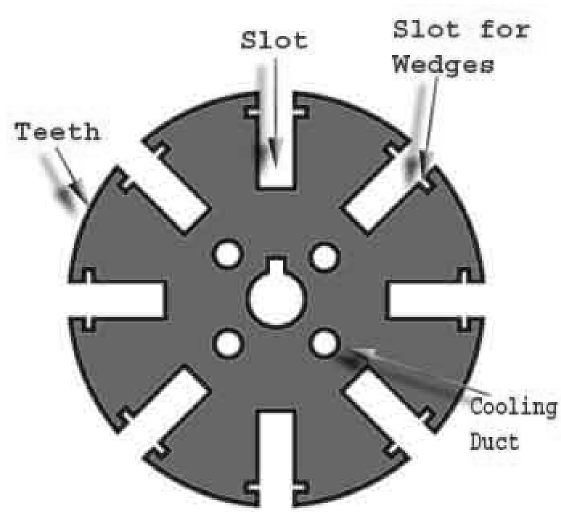




Field Winding of DC Machine

The **field winding of dc motor** are made with field coils (copper wire) wound over the slots of the pole shoes in such a manner that when field current flows through it, then adjacent poles have opposite polarity are produced. The field winding basically form an electromagnet, that produces field flux within which the rotor armature of the dc motor rotates, and results in the effective flux cutting.





Armature Winding of DC Machine

The armature winding of dc motor is attached to the rotor, or the rotating part of the machine, and as a result is subjected to altering magnetic field in the path of its rotation which directly results in magnetic losses. For this reason the rotor is made of armature core, that's made with several low-hysteresis silicon steel laminations, to reduce the magnetic losses like hysteresis and eddy current loss respectively. These laminated steel sheets are stacked together to form the cylindrical structure of the armature core.

The armature core are provided with slots made of the same material as the core to which the armature winding made with several turns of copper wire distributed uniformly over the entire periphery of the core. The slot openings a shut with fibrous wedges to prevent the conductor from plying out due to the high centrifugal force produced during the rotation of the armature, in presence of supply current and field.

The construction of **armature winding of dc motor** can be of two types:-

Lap Winding

In this case the number of parallel paths between conductors A is equal to the number of poles P.

i.e. $A = P$

***An easy way of remembering it is by remembering the word LAP-----→ L A=P

Wave Winding

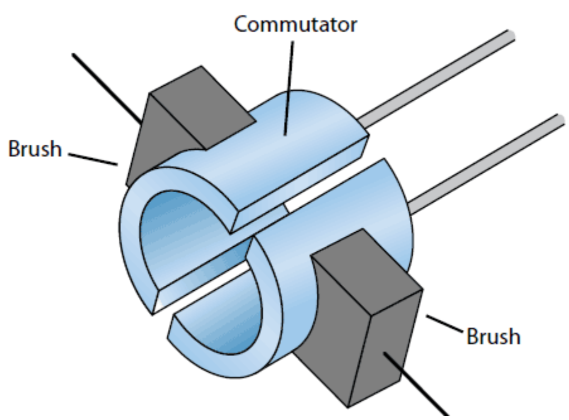
Here in this case, the number of parallel paths between conductors A is always equal to 2 irrespective of the number of poles. Hence the machine designs are made accordingly

Commutator of DC Machine

The commutator of dc motor is a cylindrical structure made up of copper segments stacked together, but insulated from each other by mica. Its main function as far as the dc motor is concerned is to commute or relay the supply current from the mains to the armature winding housed over a rotating structure through the brushes of dc motor.

Brushes of DC Machine

The brushes of dc motor are made with carbon or graphite structures, making sliding contact over the rotating commutator. The brushes are used to relay the current from external circuit to the rotating commutator form where it flows into the armature winding. So, the commutator and brush unit of the dc motor is concerned with transmitting the power from the static electrical circuit to the mechanically rotating region or the rotor.



E.M.F. Equation of a D.C. Generator

We shall now derive an expression for the e.m.f. generated in a d.c. generator.

Let $\emptyset$ = flux/pole in Wb

Z = total number of armature conductors

P = number of poles

A = number of parallel paths = 2 ... for wave winding

= P ... for lap winding

N = speed of armature in r.p.m.

E_g = e.m.f. of the generator = e.m.f./parallel path Flux cut by one conductor in one revolution of the armature,

$$d\emptyset = P\emptyset \text{ webers}$$

Time taken to complete one revolution,

$$dt = 60/N \text{ second}$$

E.M.F generated per conductor = $\frac{\emptyset P}{(60/N)} = \frac{\emptyset PN}{60}$

e.m.f. of generator,

E_g = e.m.f. per parallel path

= (e.m.f/conductor) * No. of conductors in series per parallel path

$$E_g = \frac{\emptyset PN}{60} * (Z/A)$$

$$\therefore E_g = \frac{\emptyset PNZ}{60A}$$

Where A = 2 for Wave winding

= P for lap winding

Types of DC Generators

Generally DC generators are classified according to the ways of excitation of their fields.

i) Separately excited DC generators

ii) Self excited DC generators

Separately Excited DC Generator

These are the generators whose field magnets are energized by some external dc source such as battery . A circuit diagram of separately excited DC generator is shown in figure.

I_a = Armature current

I_L = Load current

V = Terminal voltage

E_g = Generated emf

Voltage drop in the armature = $\mathbf{I_a \times R_a}$

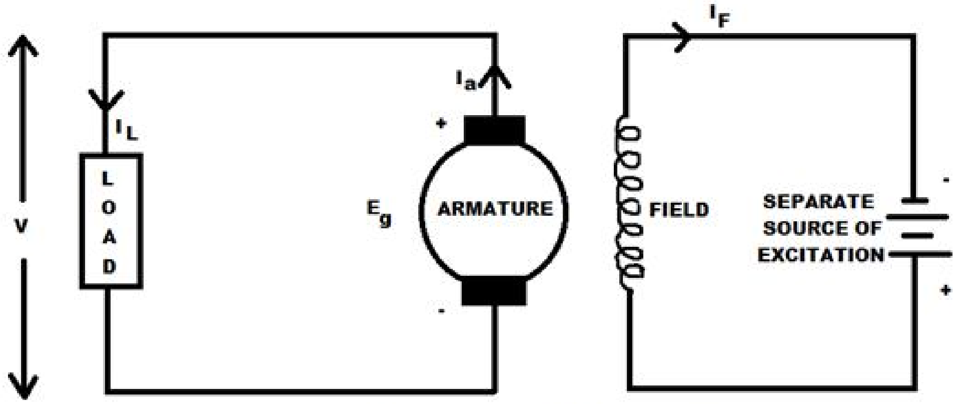
where $\mathbf{R_a}$ is the armature resistance

Let, $\mathbf{I_a = I_L = I}$ (say)

Then, voltage across the load, $\mathbf{V = IR_a}$

Power generated, $\mathbf{P_g = E_g \times I}$

Power delivered to the external load, $\mathbf{P_L = V \times I}$



Separately excited DC generator

Self-excited DC Generators

These are the generators whose field magnets are energized by the current supplied by themselves. In this type of machines field coils are internally connected with the armature. Due to residual magnetism some flux is always present in the poles. When the armature is rotated some emf is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and thereby strengthening the pole flux. As the pole flux strengthened, it will produce more armature emf, which causes further increase of current through the field. This increased field current further raises armature emf and this cumulative phenomenon continues until the excitation reaches to the rated value.

According to the position of the field coils the Self-excited DC generators may be classified as…

A. Series wound generators

B. Shunt wound generators

C. Compound wound generators

Series Wound Generator

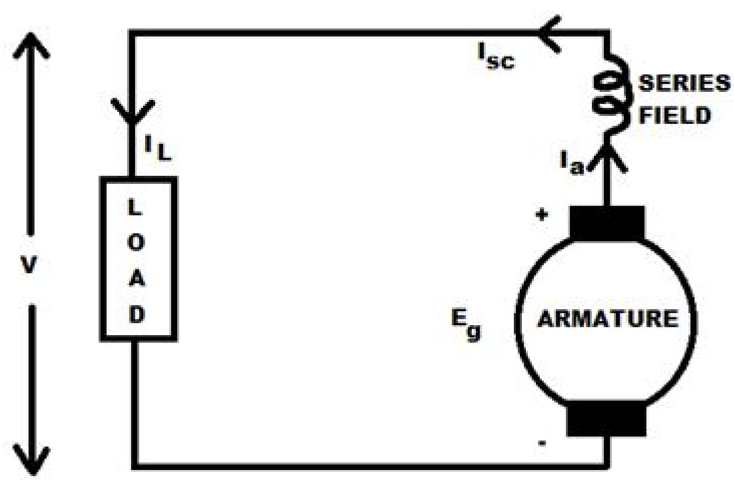
In this type of generators, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load. As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω).

Let **R_{sc}** = Series winding resistance E_g = Generated emf

I_{sc} = Current flowing through the series field **I_a** = Armature current

I_L = Load current **V**=Terminal-voltage

R_a=Armature resistance



Series Wound Generator

Then,

$I_a = I_{sc} = I_L = I$ (say)

Voltage across the load, **$V = E_g - I(I_a \times R_a)$**

Power generated, **$P_g = E_g \times I$**

Power delivered to the load, **$P_L = V \times I$**

Shunt Wound DC Generators

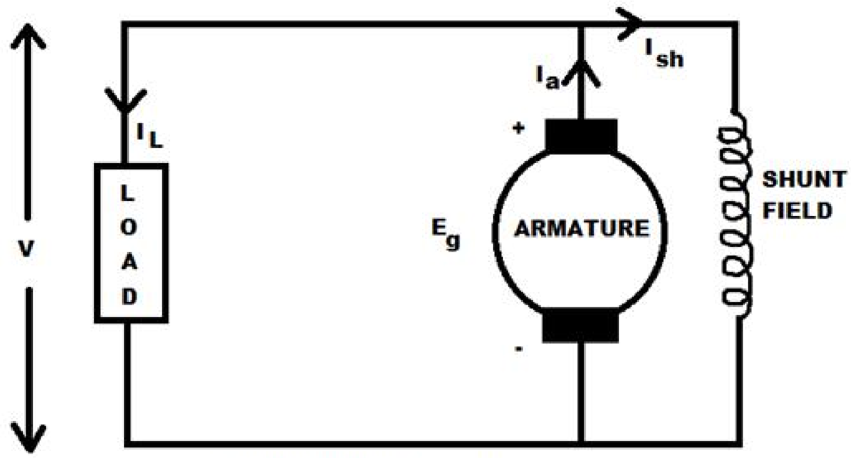
In this type of DC generators the field windings are connected in parallel with armature conductors as shown in figure below. In shunt wound generators the voltage in the field winding is same as the voltage across the terminal.

Let, $\mathbf{R_{sh}}$ = Shunt winding resistance $\mathbf{R_a}$ = Armature resistance

$\mathbf{I_{sh}}$ = Current flowing through the shunt field $\mathbf{I_a}$ = Armature current

$\mathbf{I_L}$ = Load current $\mathbf{E_g}$ = Generated emf

V = Terminal voltage



Shunt Wound Generator

Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L. So, $I_a=I_{sh} + I_L$ The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf.

Shunt field current, $\mathbf{I_{sh} = V/R_{sh}}$

Voltage across the load, $\mathbf{V = E_g - I_a R_a}$

Power generated, $\mathbf{P_g = E_g \times I_a}$

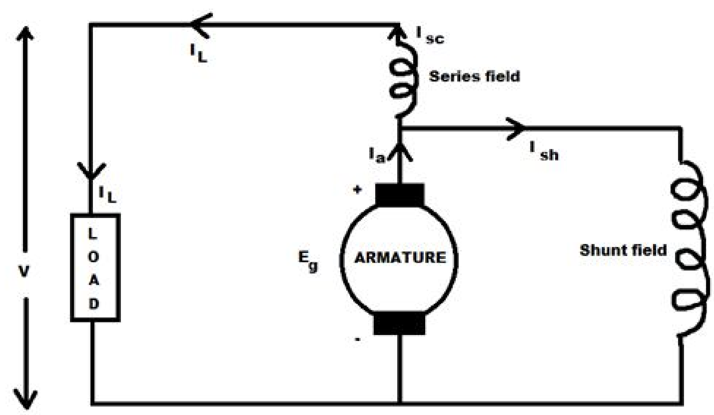
Power delivered to the load, $\mathbf{P_L = V \times I_L}$

Compound Wound DC Generator

In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current. A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator. Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature. This type of DC generators may be of two types- short shunt compound wound generator and long shunt compound wound generator.

Short Shunt Compound Wound DC Generator

The generator in which only shunt field winding is in parallel with the armature winding as shown in below figure



Short Shunt Compound Wound Generator

Series field current, $I_{sc} = I_L$

Shunt field current, $I_{sh} = (V+I_{sc} R_{sc})/R_{sh}$

Armature current, $I_a = I_{sh} + I_L$

Voltage across the load, $V = E_g - I_a R_a - I_{sc} R_{sc}$

Power generated, $P_g = E_g \times I_a$

Power delivered to the load, $P_L = V \times I_L$

Long Shunt Compound Wound DC Generator :

The generator in which shunt field winding is in parallel with both series field and armature winding as shown in figure.

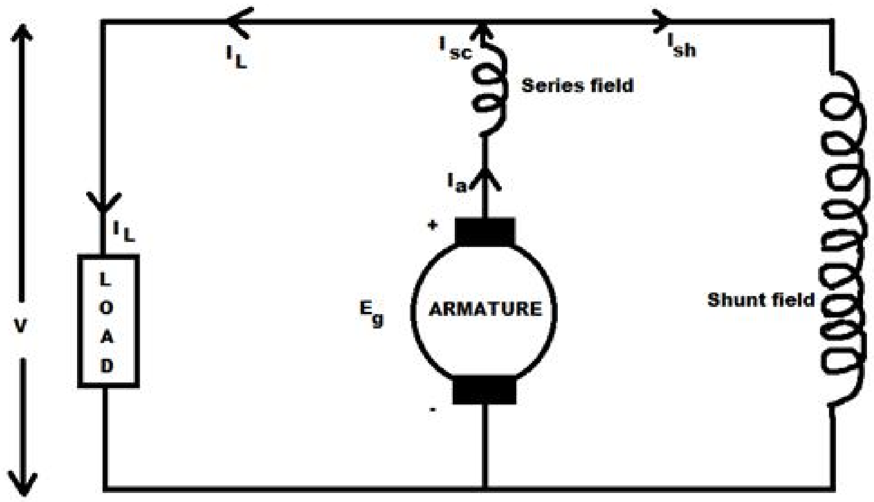
Shunt field current, $\mathbf{I_{sh}=V/R_{sh}}$

Armature current, $\mathbf{I_a}$= series field current, $\mathbf{I_{sc}= I_L+I_{sh}}$

Voltage across the load, $\mathbf{V=E_g-I_a R_a-I_{sc} R_{sc}=E_g-I_a (R_a+R_{sc})}$ $[\because \mathbf{I_a=I_{sc}}]$

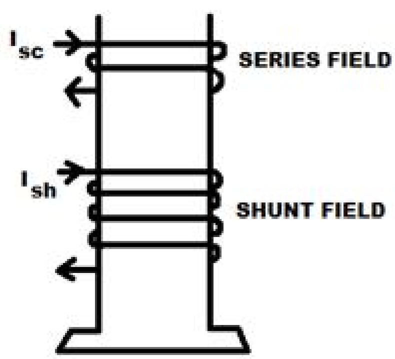
Power generated, $\mathbf{P_g= E_g \times I_a}$

Power delivered to the load, $\mathbf{P_L=V \times I_L}$

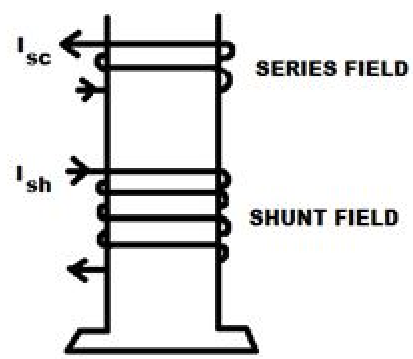


Long Shunt Compound Wound Generator

In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand if series field opposes the shunt field, the generator is said to be differentially compound wound.



CUMULATIVE COMPOUNDING



DIFFERENTIAL COMPOUNDING

Applications of DC Generators:

There are various types of DC generators available for several types of services. The applications of these dc generators based on their characteristic are discussed below:

Applications of Separately Excited DC Generators:

These types of DC generators are generally more expensive than self-excited DC generators because of their requirement of separate excitation source. Because of that their applications are restricted. They are generally used where the use of self-excited generators are unsatisfactory.

I. Because of their ability of giving wide range of voltage output, they are generally used for testing purpose in the laboratories.

II. Separately excited generators operate in a stable condition with any variation in field excitation. Because of this property they are used as supply source of DC motors, whose speeds are to be controlled for various applications. Example- Ward Leonard Systems of speed control.

Applications of Shunt Wound DC Generators

The applications of shunt generators are very much restricted for its dropping voltage characteristic. They are used to supply power to the apparatus situated very close to its position. These type of DC generators generally give constant terminal voltage for small distance operation with the help of field regulators from no load to full load.

I. They are used for general lighting.

II. They are used to charge battery because they can be made to give constant output voltage.

III. They are used for giving the excitation to the alternators.

IV. They are also used for small power supply.

Applications of Series Wound DC Generators

These types of generators are restricted for the use of power supply because of their increasing terminal voltage characteristic with the increase in load current from no load to full load. We can clearly see this characteristic from the characteristic curve of series wound generator. They give constant current in the dropping portion of the characteristic curve. For this property they can be used as constant current source and employed for various applications.

I. They are used for supplying field excitation current in DC locomotives for regenerative breaking.

II. This types of generators are used as boosters to compensate the voltage drop in the feeder in various types of distribution systems such as railway service.

III. In series arc lightening this type of generators are mainly used.

Applications of Compound Wound DC Generators:

Among various types of DC generators, the compound wound DC generators are most widely used because of its compensating property. We can get desired terminal voltage by compensating the drop due to armature reaction and ohmic drop in the in the line. Such generators have various applications.

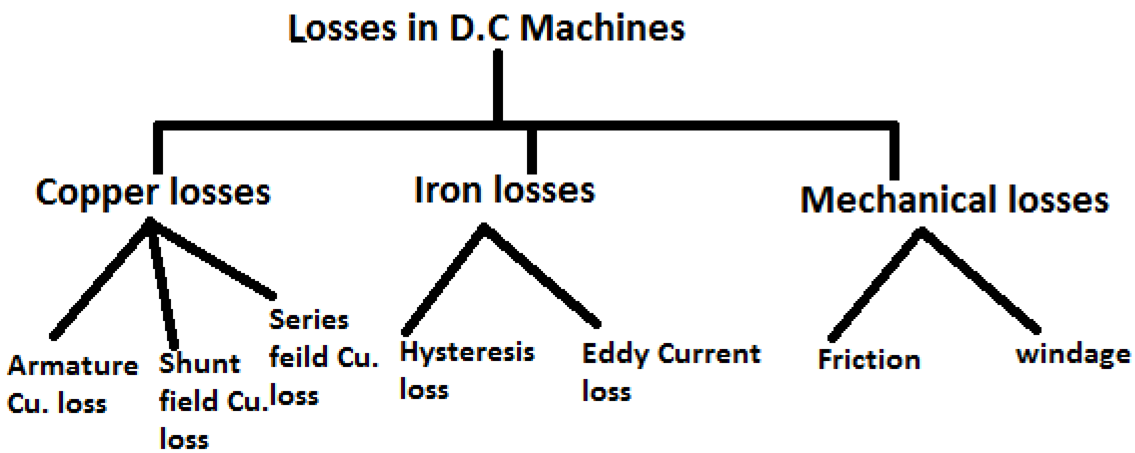
I. Cumulative compound wound generators are generally used lighting, power supply purpose and for heavy power services because of their constant voltage property. They are mainly made over compounded.

II. Cumulative compound wound generators are also used for driving a motor.

III. For small distance operation, such as power supply for hotels, offices, homes and lodges, the flat compounded generators are generally used.

IV. The differential compound wound generators, because of their large demagnetization armature reaction, are used for arc welding where huge voltage drop and constant current is required.

Power losses in a DC machine:



The **losses in a dc machine (generator or motor)** may be divided into three classes. They are

(i) Copper losses

(ii) Iron or core losses and

(iii) Mechanical losses.

All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.

Copper losses

These losses occur due to currents in the various windings of the machine.

1. Armature copper loss = $I_a^2R_a$
2. Shunt field copper loss = $I_{sh}^2R_{sh}$
3. Series field copper loss = $I_{se}^2R_{se}$

Note. There is also brush contact loss due to brush contact resistance (i.e., resistance between the surface of brush and surface of commutator). This loss is generally included in armature copper loss.

Iron or Core losses

These losses occur in the armature of a D.C. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

(i) Hysteresis loss

Hysteresis loss occurs in the armature of the d.c. machine since any given part of the armature is subjected to magnetic field reversals as it passes under successive poles.

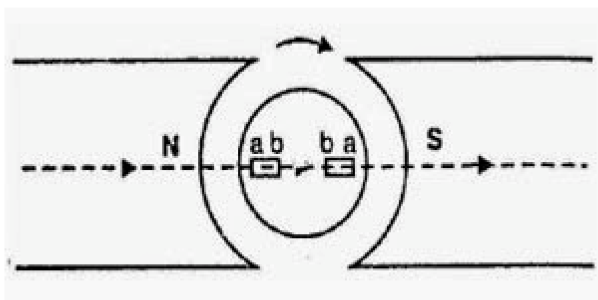


Figure shows an armature rotating in two-pole machine. Consider a small piece ab of the armature. When the piece ab is under N-pole, the magnetic lines pass from a to b. Half a revolution later, the same piece of iron is under S-pole and magnetic lines pass from b to a so that magnetism in the iron is reversed. In order to reverse continuously the molecular magnets in the armature core, some amount of power has to be spent which is called hysteresis loss. It is given by Steinmetz formula. This formula is

Hysteresis loss $P_h = \eta B_{max}^{1.6} f V$

Where Bmax = Maximum flux density in armature

f = Frequency of magnetic reversals

= NP/120 where N is in r.p.m

V = Volume of armature in m3

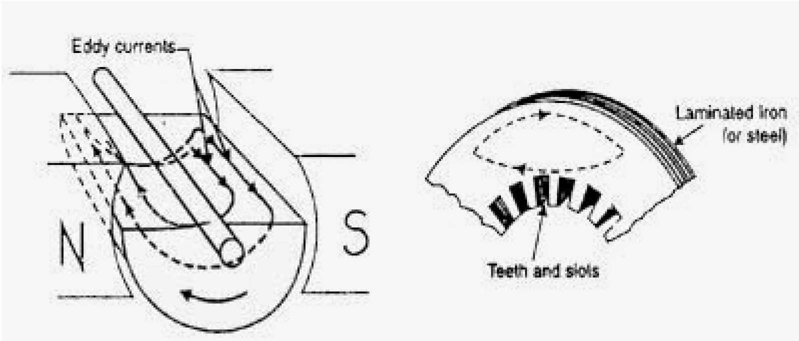
h = Steinmetz hysteresis co-efficient

In order to reduce this loss in a d.c. machine, armature core is made of such materials which have a low value of Steinmetz hysteresis co-efficient e.g. silicon steel.

(ii) Eddy current loss

In addition to the voltages induced in the armature conductors, there are also voltages induced in the armature core. These voltages produce circulating currents in the armature core as shown in Fig. These are called eddy currents and power loss due to their flow is called eddy current loss. The eddy current loss appears as heat which raises the temperature of the machine and lowers its efficiency.

If a continuous solid iron core is used, the resistance to eddy current path will be small due to large cross-sectional area of the core. Consequently, the magnitude of eddy current and hence eddy current loss will be large. The magnitude of eddy current can be reduced by making core resistance as high as practical. The core resistance can be greatly increased by constructing the core of thin, round iron sheets called laminations. The laminations are insulated from each other with a coating of varnish. The insulating coating has a high resistance, so very little current flows from one lamination to the other. Also, because each lamination is very thin, the resistance to current flowing through the width of a lamination is also quite large. Thus laminating a core increases the core resistance which decreases the eddy current and hence the eddy current loss.



Eddy current loss $P_e = K_e B_{max} f^2 t^2 V$

Where Ke = Constant depending upon the electrical resistance of core and system of units used

Bmax = Maximum flux density in Wb/m_2

f = Frequency of magnetic reversals in Hz

t = Thickness of lamination in m

V = Volume of core in m_3

It may be noted that eddy current loss depends upon the square of lamination thickness. For this reason, lamination thickness should be kept as small as possible.

Mechanical losses

These losses are due to friction and windage.

(i) Friction loss e.g., bearing friction, brush friction etc.

(ii) windage loss i.e., air friction of rotating armature.

These losses depend upon the speed of the machine. But for a given speed, they are practically constant.

Note. Iron losses and mechanical losses together are called **stray losses**.

Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

(i) Constant losses Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

(a) iron losses

(b) mechanical losses

(c) shunt field losses

(ii) Variable losses

Those losses in a d.c. generator which vary with load are called variable losses. The variable losses in a d.c. generator are:

(a) Copper loss in armature winding ($I^2_a R_a$)

(b) Copper loss in series field winding ($I_{se}^2 R_{se}$)

Total losses = Constant losses + Variable losses

Condition for Maximum Efficiency:

The efficiency of a d.c. generator is not constant but varies with load. Consider a shunt generator delivering a load current I_L at a terminal voltage V.

Generator output = V I_L

Generator input = Output + Losses

= V I_L + Variable losses + Constant losses

= $VI_L + I_a^2 R_a + W_C$

$= VI_L + (I_L + I_{sh})^2 R_a + W_c \qquad (since\ I_a = I_L + I_{sh})$

The shunt field current I_{sh} is generally small as compared to I_L and therefore, it can be neglected.

Generator input = $VI_L + I^2_L R_a + W_C$

$$\eta = \frac{output}{input} = \frac{VI_L}{VI_L + {I_L}^2 R_a + W_c}$$

$$= \frac{1}{1 + \left[\frac{I_L R_a}{V} + \frac{W_c}{VI_L}\right]}$$

The efficiency will be maximum when the denominator of above Equation is minimum i.e.,

$$\frac{d}{dI_L}\left[\frac{I_L R_a}{V} + \frac{W_c}{VI_L}\right] = 0$$

$$\frac{R_a}{V} + \frac{W_c}{VI_L{}^2} = 0$$

$$or\ \frac{R_a}{V} = \frac{W_c}{VI_L{}^2}$$

$$or\ I_L{}^2R_a = W_c$$

Hence Variable loss = Constant loss

The load current corresponding to maximum efficiency is given by;

$$I_L = \sqrt{\frac{W_c}{R_a}}$$

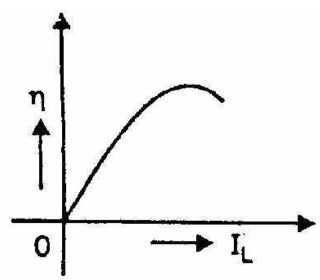


Fig 2.22 Efficiency curve of a DC generator

UNIT III
Assignment Cum Tutorial

Section - A

Objective Questions

1) The armature of a DC machine is laminated to reduce [ ]
   a) Eddy current Loss b) Hysteresis loss c) Copper loss d) Friction and windage loss
2) The Field winding of a self excited dc generator is excited by [ ]
   a) DC b) AC c) Either a or b d) its own current.
3) The emf induced in armature of dc generator is [ ]
   a) DC b) AC c) Either a or b d) both.
4) The direction of induced emf in generator is given by [ ]
   a) Fleming Right hand rule b) Fleming Left hand rule
   c) Fleming Right hand screw rule d) none
5) HP rating on name plate of a DC motor indicates [ ]
   a) output power b) input power c) both d) any of these
6) Commutator in DC generator is used for [ ]
   a) collecting of current b) reduce losses
   c) increase efficiency d) convert AC armature current in to DC
7) A DC generator without commutator is a [ ]
   a) AC generator b) DC motor c) DC generator d)induction motor
8) Series generators are used in which of the following applications [ ]
   a) air crafts b) arc welding
   c) Used as boosters in dc distribution or transmission d) all of the above
9) Which of the following generators are used in arc welding? [ ]
   a) Shunt generators b) series generators
   c) Cumulative compound generators d) differential compound generators
10) The armature of DC generator is laminated to [ ]
   a) Reduce the bulk b) provide the bulk

c) Insulate the core d) reduce eddy current loss

11) A P pole lap wound dc machine had an armature current I. The conductor current in the armature winding is [ ]

a) I b) I/P c) PI d) none of the above.

12) A shunt generator running at 1000 rpm, if flux is reduced by half, then what is the new speed? [ ]

a) 1000 b) 2000 c) 500 d) 0

13) A Shunt generator running at 1000 rpm has generated emf of 100 V. If the speed increases to 1200 rpm, the generated emf will be nearly [ ]

a) 120 V b) 140 V c) 175 V d) 240 V

14) Which of the following is not true for Yoke? [ ]

a) It carries field winding b) It provides Mechanical support for field poles

c) It provides path for filed flux d) It is made up of cast steel.

15) A dc 4 pole lap wound generator is running at 1000 rpm having 1200 conductors and flux density is 10 mWb. Find the generated emf? [ ]

a) 20V b) 10V c) 200V d) 100V

16) A 4 pole dc generator is running at 1500 rpm the frequency of current in the armature winding is [ ]

a) 50Hz b) 100Hz c) 150Hz d) 200Hz

17) An 8 pole DC generator has 500 armature conductors and useful per pole of 0.065 Wb. What will be the emf generated if it is lap connected and runs at 1000 rpm. What must be the speed at which it is driven to produce the same emf if it is wave wound? [ ]

a) 200 rpm b) 230 rpm c) 250 rpm d) 270 rpm

18) A lap wound DC machine has 400 conductors and 8 poles. The voltage induced per conductor is 2 volts. The machine generates a voltage of

a) 100 V b) 200 V c) 400 V d) 800 V[ ]

19) A 4 pole dynamo with wave wound armature has 51 slots contains 20 conductors in each slot. If induced emf 357 Volts and speed is 8500 rpm, flux per pole will be [ ]

a) 1.2 mWb b) 1.4 mWb c) 2.7 mWb d) 3.5 mWb

20) In a DC generator, if P be the number of poles and N be the rpm of the rotor, then frequency of magnetic reversals will be [ ]

a) NP / 2 b) NP / 60 c) NP / 120 d) NP / 3000

21) An 8-pole, DC generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 Wb. The machine is running at 250 rpm. The induced armature voltage is [ ]

(a) 96 V (b) 192 V (c) 384 V (d) 768 V

Section – B

Descriptive Questions

1) Explain with a neat sketch the constructional details of a DC machine.
2) Explain the basic principle of operation of a dc generator.
3) Develop from first principles, an expression for the EMF Equation of a DC machine.
4) Explain the classification of dc generators with neat circuit diagrams. Also write the relationships among the currents and voltages.
5) Explain about the different losses that occur in a D.C machine. How these are minimized?
6) What are the applications of DC generators?
7) A 30 kW 300V DC shunt generator has armature and field resistance of 0.05 Ω and 100 Ω respectively. Calculate total power developed by the armature when it delivers full load output.
8) A 4-pole generator has a flux of 40mWb per pole and a lap connected armature with 740 conductors. Determine the emf generated at 1000 rpm.
9) A 25-KW, 250V, D.C. shunt generator has armature and field resistances of 6Ω and 100Ω respectively. Determine the total armature power developed when working as a generator delivering 25KW output.
10) A 6-pole machine has an armature with 90 slots and 8 conductors per

slot and runs at 1000 rpm, the flux per pole is 0.05wb. Determine the induced emf if winding is (i) Lap connected and (ii) Wave connected.

11) A 200V DC shunt machine has an armature resistance of 0.5Ω.if the full load armature current is 20A. Find the induced emf when the machine acts as generator.

12) An 8 pole dc shunt generator has 778 wave connected armature conductors running at 500 r.p.m. supplies a load of 12.5Ω resistance at a terminal voltage of 250v. The armature resistance is 0.24Ω and the field resistance is 250Ω .Find out the armature current, the induced emf and the flux per pole.

13) A short shunt compound generator supplied 7.5Kw at 230V. The shunt field, series field and armature resistances are 100Ω, 0.3Ω & 0.4Ω respectively. Calculate the induced emf and the load resistance.

14) A 6 pole DC generator has 250 armature conductors and useful per pole of 0.065 Wb. What will be the emf generated if it is lap connected and runs at 1500 rpm. What must be the speed at which it is driven to produce the same emf if it is wave wound?

Section – C

GATE / IES QUESTONS

1) A DC shunt generator delivers 45 A at a terminal voltage of 220 V. The armature and the shunt field resistances are 0.01 Ω and 44 Ω respectively. The stray losses are 375 W. The percentage efficiency of the DC generator is ___________. **GATE-2016**

2) A separately excited DC generator has an armature resistance of 0.1ΩΩ and negligible armature inductance. At rated field current and rated rotor speed, its open-circuit voltage is 200 V. When this generator is operated at half the rated speed, with half the rated field current, an un-charged 1000 μF capacitor is suddenly connected across the armature terminals. Assume that the speed remains unchanged during the transient. At what time (in microsecond) after the capacitor is connected will the voltage across it reach 25 V?

GATE-2015

(A) 62.25 (B) 69.3 (C) 73.25 (D) 77.3 [ ]

3) A 4-pole, separately excited, wave wound DC machine with negligible armature resistance is rated for 230 V and 5 kW at a speed of 1200 rpm. If the same armature coils are reconnected to forms a lap winding, what is the rated voltage (in volts) and power (in kW) respectively at 1200 rpm of the reconnected machine if the field circuit is left unchanged? **GATE-2015**

(A) 230 and 5 (B) 115 and 5 (C) 115 and 2.5 (D) 230 and 2.5

4) A 250 V dc shunt machine has armature circuit resistance of 0.6 Ω and field circuit resistance of 125 Ω. The machine is connected to 250 V supply mains. The motor is operated as a generator and then as a motor separately. The line current of the machine in both the cases is 50 A. The ratio of the speed as a generator to the speed as a motor is ___________. **GATE-2014**

5) A 220 V DC machine supplies 20 A at 200 V as a generator. The armature resistance is 0.2 ohm. If the machine is now operated as a motor at same terminal voltage and current but with the flux increased by 10%, the ratio of motor speed to generator speed is **GATE-2006**

(A) 0.87 (B) 0.95 (C) 0.96 (D) 1.06 [ ]

6) In a DC machine, which of the following statements is true? **GATE-2006**

(A) Compensating winding is used for neutralizing armature reaction while interpole winding is used for producing residual flux [ ]

(B) Compensating winding is used for neutralizing armature reaction while interpole winding is used for improving commutation

(C) Compensating winding is used for improving commutation while interpole winding is used for neutralizing armature reaction

(D) Compensation winding is used for improving commutation while interpole winding is used for producing residual flux

7) A 8-pole, DC generator has a simplex wave-wound armature containing 32 coils of 6 turns each. Its flux per pole is 0.06 Wb. The machine is running at 250 rpm. The induced armature voltage is **GATE-2004**

(A) 96 V (B) 192 V (C) 384 V (D) 768 V [ ]

8) The dummy coils in the DC machine are useful to **IES - 2003**

a) Increase the efficiency of the machine [ ]

b) Improve the commutation

c) Reduce the armature reaction

d) Maintain mechanical balance

9) A dc shunt generator is supplying a load of 1.8 kW at 200 V. Its armature and field resistances are 0 4. Ω and 200 Ω respectively. What is the generated emf? **IES - 2007**

(A) 190 V (B) 196 V (C) 204 V (D) 210 V[ ]

10) A self-excited d.c. shunt generator, driven by its prime-mover at the rated speed fails to build up voltage across its terminals at no-load. What reason can be assigned for this? **IES - 2006**

(A) The field circuit resistance is higher than the critical resistance[ ]

(B) The initial shunt field mmf does not assist the residual magnetism

(C) One of the inter-pole connections is reversed

(D) The brush-axis shifts slightly from the geometrical neutral axis of the machine

11) Wave winding is employed in a d.c. machine of **IES - 2006**

(A) high current and low voltage rating [ ]

(B) low current and high voltage rating

(C) high current and high voltage rating

(D) low current and low voltage rating

UNIT -IV

DC MOTORS

Objectives:

- To familiarize students with the principle of operation of DC Motors
- To impart some basic knowledge on types of motors, losses and torque equations.

Syllabus:

DC Motors

Construction & Principle of operation of DC Motors — types of DC Motors – Losses and torque equation. (All the above topics are only elementary treatment and simple problems)

Learning Outcomes:

Student will be able to

1. Understand the principle of operation of DC Motors.
2. Learn the torque equation and losses occurring in the machine.
3. Solve the problems on DC Motors

D.C MOTORS

Faraday's Laws of Electromagnetic Induction:

In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's laws of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors . This law shows the relationship between electric circuit and magnetic field. Faraday performs an experiment with a magnet and coil. During this experiment, he found how emf is induced in the coil when flux linked with it changes. He has also done experiments in electro-chemistry and electrolysis.

Faraday's First Law

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current. Method to change magnetic field:

1. By moving a magnet towards or away from the coil
2. By moving the coil into or out of the magnetic field.
3. By changing the area of a coil placed in the magnetic field
4. By rotating the coil relative to the magnet.

Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Consider a magnet approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time, $T_1 = N\Phi_1$ Wb

Flux linkage with the coil at time, $T_2 = N\Phi_2$ wb

Change in flux linkage = $N(\Phi_2 - \Phi_1)$

Let this change in flux linkage be, $\Phi = \Phi_2 - \Phi_1$

So, the Change in flux linkage = $N\Phi$

Now the rate of change of flux linkage = $N\Phi / t$

Take derivative on right hand side we will get

The rate of change of flux linkage = $Nd\Phi/dt$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = N\frac{d\emptyset}{dt}$$

Considering Lenz's Law.

$$E = -N\frac{d\emptyset}{dt}$$

Where flux Φ in wb = B.A

B = magnetic field strength

A = area of the coil

Lenz's Law

Lenz's law states that when an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such, that it produces an current that's magnetic field opposes the change which produces it.

The negative sign used in Faraday's law of electromagnetic induction, indicates that the induced emf (E) and the change in magnetic flux ($d\Phi_B$) have opposite signs.

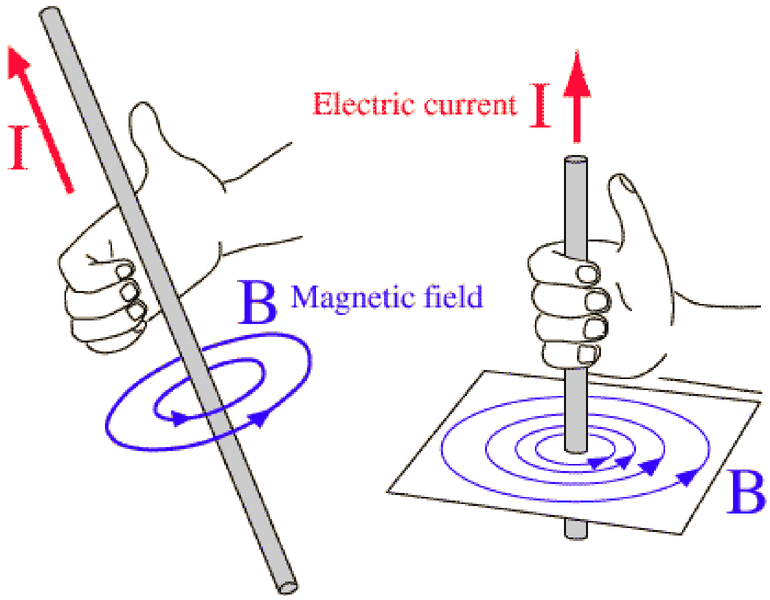
$$E = -N\frac{d\emptyset_B}{dt}$$

Where E = Induced emf

$d\Phi_B$ = change in magnetic flux

N = No of turns in coil

NOTE : For finding the directions of magnetic field or current, use right hand thumb rule i.e. if the fingers of the right hand are placed around the wire so that the thumb points in the direction of current flow, then the curling of fingers will show the direction of the magnetic field produced by the wire.



Construction of DC Machine (for both Generator and Motor)

A DC motor like we all know is a device that deals in the conversion of electrical energy to mechanical energy and this is essentially brought about by two major parts required for the **construction of dc motor**, namely.

1) Stator – The static part that houses the field windings and receives the supply and,

2) Rotor – The rotating part that brings about the mechanical rotations.

3) Yoke of dc motor.

4) Poles of dc motor.

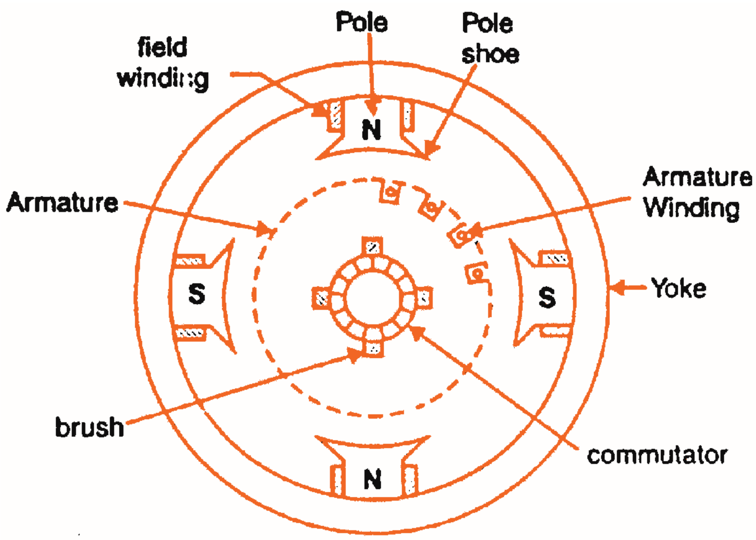
5) Field winding of dc motor.

6) Armature winding of dc motor.

7) Commutator of dc motor.

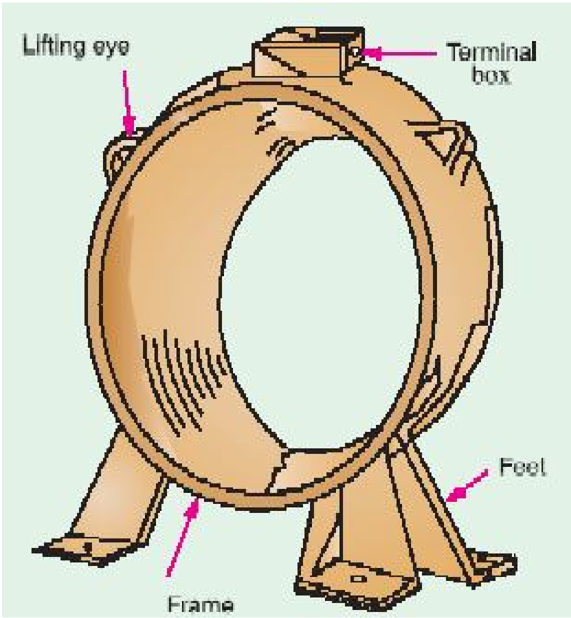
8) Brushes

All these parts put together configures the total **construction of a dc motor**. Now let's do a detailed discussion about all the essential parts of dc motor.



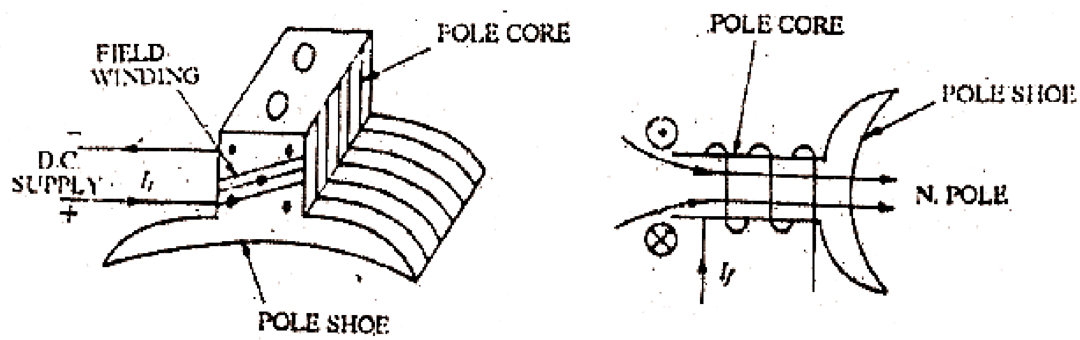
Yoke of DC Machine

The magnetic frame or the **yoke of dc motor** made up of cast iron or steel and forms an integral part of the stator or the static part of the motor. Its main function is to form a protective covering over the inner sophisticated parts of the motor and provide support to the armature. It also supports the field system by housing the magnetic poles and field winding of the dc motor.



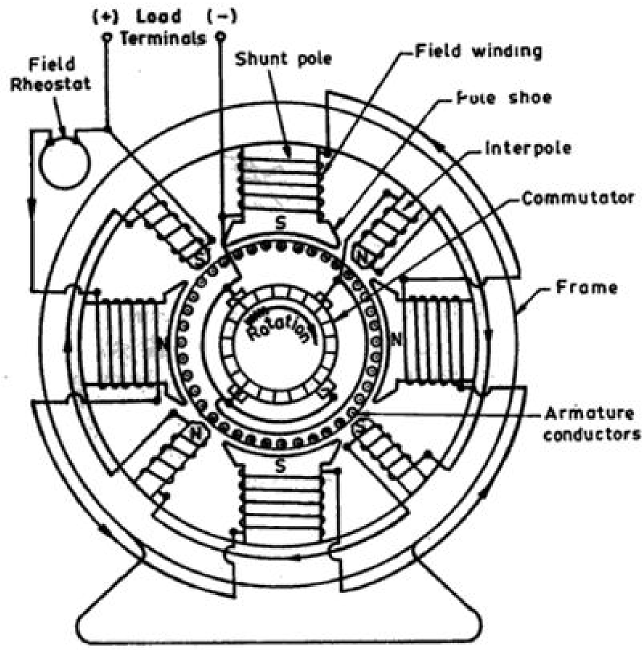
Poles of DC Machine

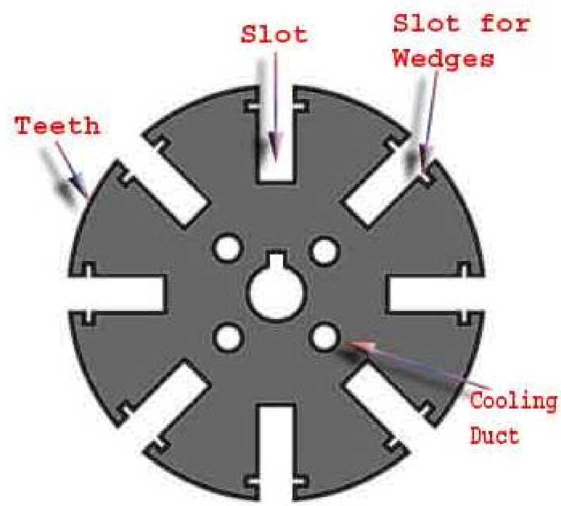
The magnetic **poles of DC motor** are structures fitted onto the inner wall of the yoke with screws. The construction of magnetic poles basically comprises of two parts namely, the pole core and the pole shoe stacked together under hydraulic pressure and then attached to the yoke. These two structures are assigned for different purposes, the pole core is of small cross sectional area and its function is to just hold the pole shoe over the yoke, whereas the pole shoe having a relatively larger cross-sectional area spreads the flux produced over the air gap between the stator and rotor to reduce the loss due to reluctance. The pole shoe also carries slots for the field windings that produce the field flux.



Field Winding of DC Machine

The **field winding of dc motor** are made with field coils (copper wire) wound over the slots of the pole shoes in such a manner that when field current flows through it, then adjacent poles have opposite polarity are produced. The field winding basically form an electromagnet, that produces field flux within which the rotor armature of the dc motor rotates, and results in the effective flux cutting.





Armature Winding of DC Machine

The armature winding of dc motor is attached to the rotor, or the rotating part of the machine, and as a result is subjected to altering magnetic field in the path of its rotation which directly results in magnetic losses. For this reason the rotor is made of armature core, that's made with several low-hysteresis silicon steel laminations, to reduce the magnetic losses like hysteresis and eddy current loss respectively. These laminated steel sheets are stacked together to form the cylindrical structure of the armature core.

The armature core are provided with slots made of the same material as the core to which the armature winding made with several turns of copper wire distributed uniformly over the entire periphery of the core. The slot openings a shut with fibrous wedges to prevent the conductor from plying out due to the high centrifugal force produced during the rotation of the armature, in presence of supply current and field.

The construction of **armature winding of dc motor** can be of two types:-

Lap Winding

In this case the number of parallel paths between conductors A is equal to the number of poles P.

i.e. A = P

***An easy way of remembering it is by remembering the word LAP-----→ L

A=P

Wave Winding

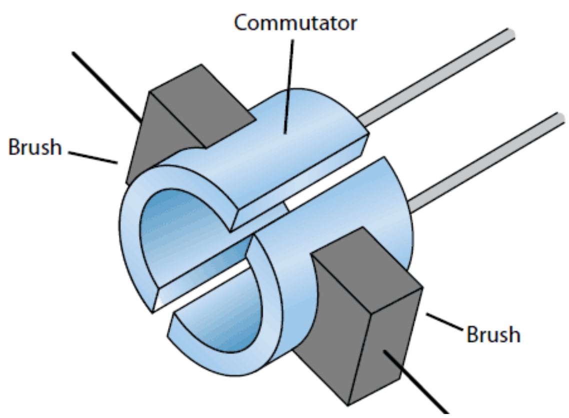
Here in this case, the number of parallel paths between conductors A is always equal to 2 irrespective of the number of poles. Hence the machine designs are made accordingly

Commutator of DC Machine

The commutator of dc motor is a cylindrical structure made up of copper segments stacked together, but insulated from each other by mica. Its main function as far as the dc motor is concerned is to commute or relay the supply current from the mains to the armature winding housed over a rotating structure through the brushes of dc motor.

Brushes of DC Machine

The brushes of dc motor are made with carbon or graphite structures, making sliding contact over the rotating commutator. The brushes are used to relay the current from external circuit to the rotating commutator form where it flows into the armature winding. So, the commutator and brush unit of the dc motor is concerned with transmitting the power from the static electrical circuit to the mechanically rotating region or the rotor.



ELECTRICAL MOTOR

The motor or an electrical motor is a device that has brought about one of the biggest advancements in the fields of engineering and technology ever since the invention of electricity. A motor is nothing but an electro-mechanical device that converts electrical energy to mechanical energy. It's because of motors, life is what it is today in the 21st century. Without motor we had still been living in Sir Thomas Edison's Era where the only purpose of electricity would have been to glow bulbs. There are different types of motors have been developed for different specific purposes.

In simple words we can say a device that produces rotational force is a motor. The very basic principal of functioning of an **electrical motor** lies on the fact that force is experienced in the direction perpendicular to magnetic field and the current, when field and current are made to interact with each other. Ever since the invention of motors, a lot of advancements have taken place in this field of engineering and it has become a subject of extreme importance for modern engineers.

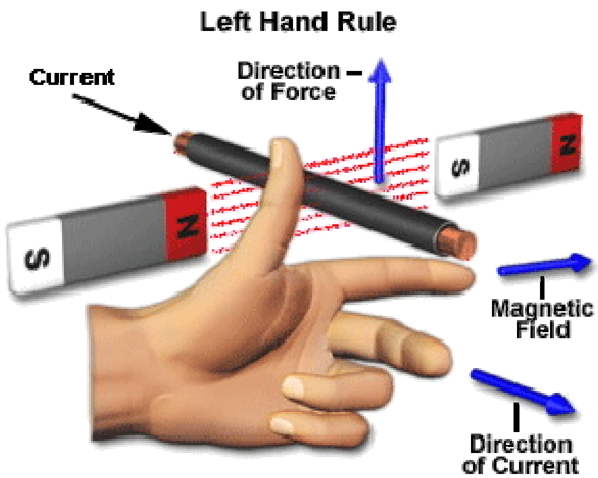
DC Motor

Electrical motors are everywhere around us. Almost all the electro-mechanical movements we see around us are caused either by an A.C. or a **DC motor**. Here we will be exploring this kind of motors. This is a device that converts DC electrical energy to a mechanical energy.

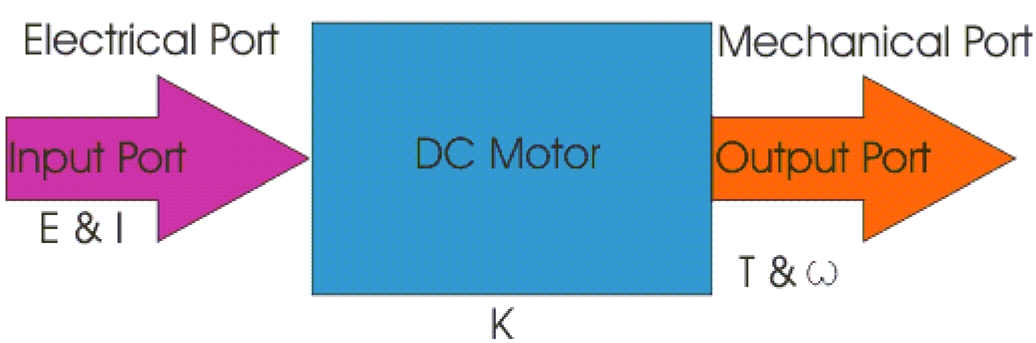
Principle of DC Motor

This DC or **direct current motor** works on the principal, when a current carrying conductor is placed in a magnetic field, it experiences a torque and has a tendency to move. This is known as motoring action. If the direction of current in the wire is reversed, the direction of rotation also reverses. When magnetic field and electric field interact they produce a mechanical force, and based on that the working principle of **dc motor** established.

The direction of rotation of a this motor is given by Fleming's left hand rule, which states that if the index finger, middle finger and thumb of your left hand are extended mutually perpendicular to each other and if the index finger represents the direction of magnetic field, middle finger indicates the direction of current, then the thumb represents the direction in which force is experienced by the shaft of the **dc motor**.



Structurally and construction wise a direct current motor is exactly similar to a DC generator, but electrically it is just the opposite. Here we unlike a generator we supply electrical energy to the input port and derive mechanical energy from the output port. We can represent it by the block diagram shown below.



Here in a DC motor, the supply voltage E and current I is given to the electrical port or the input port and we derive the mechanical output i.e. torque T and speed ω from the mechanical port or output port.

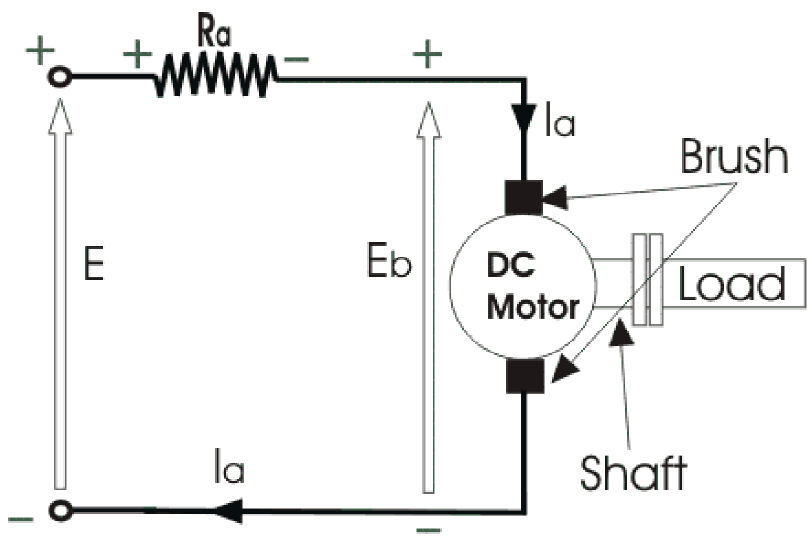
The input and output port variables of the **direct current motor** are related by the parameter K.

$$T = KI \text{ and } E = K\omega$$

So from the picture above we can well understand that motor is just the opposite phenomena of a DC generator, and we can derive both motoring and generating operation from the same machine by simply reversing the ports.

Detailed Description of a DC Motor

To understand the DC motor in details lets consider the diagram below,



The direct current motor is represented by the circle in the center, on which is mounted the brushes, where we connect the external terminals, from where supply voltage is given. On the mechanical terminal we have a shaft coming out of the Motor, and connected to the armature, and the armature-shaft is coupled to the mechanical load. On the supply terminals we represent the armature resistance R_a in series. Now, let the input voltage E, is applied across the brushes. Electric current which flows through the rotor armature via brushes, in presence of the magnetic field, produces a torque T_g . Due to this torque T_g the dc motor armature rotates. As the armature conductors are carrying currents and the armature rotates inside the stator magnetic field, it also produces an emf E_b in the manner very similar to that of a generator. The generated Emf E_b is directed opposite to the supplied voltage and is known as the back Emf, as it counters the forward voltage. The back emf like in case of a generator is represented by

$$\mathbf{E_b} = \frac{\varnothing PNZ}{60A} \text{..............................} (1)$$

Where, P = no of poles

φ = flux per pole

Z= No. of conductors

A = No. of parallel paths

and N is the speed of the DC Motor.

So from the above equation we can see E_b is proportional to speed 'N'. That is whenever a direct current motor rotates it results in the generation of back Emf. Now lets represent the rotor speed by ω in rad/sec. So E_b is proportional to ω.

So when the speed of the motor is reduced by the application of load, E_b decreases. Thus the voltage difference between supply voltage and back emf increases that means $E - E_b$ increases. Due to this increased voltage difference, armature current will increase and therefore torque and hence speed increases. Thus a DC Motor is capable of maintaining the same speed under variable load.

Now armature current I_a is represented by

$$I_a = \frac{E - E_b}{R_a}$$

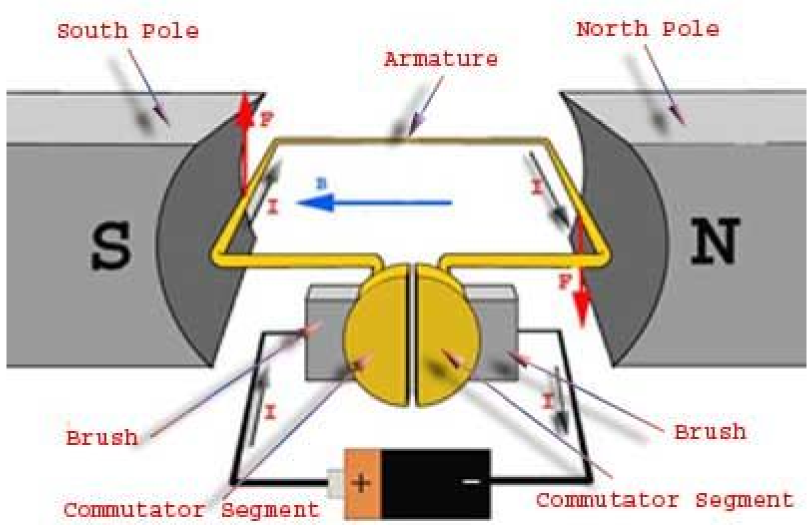
Now at starting, speed ω = 0 so at starting E_b = 0.

$$\therefore \mathbf{I_a} = \frac{\mathbf{E}}{\mathbf{R_a}} \text{ ....} (2)$$

Now since the armature winding electrical resistance R_a is small, this motor has a very high starting current in the absence of back Emf. As a result we need to use a starter for starting a DC Motor.
Now as the motor continues to rotate, the back Emf starts being generated and gradually the current decreases as the motor picks up speed.

Working or Operating Principle of DC Motor

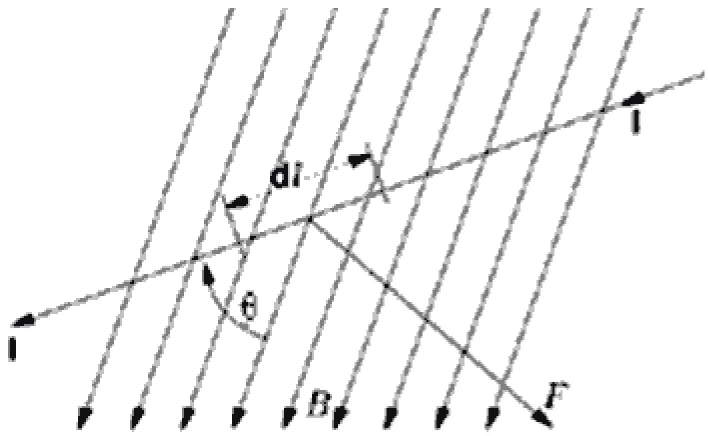
A DC motor in simple words is a device that converts direct current (electrical energy) into mechanical energy. It's of vital importance for the industry today, and is equally important for engineers to look into the **working principle of DC motor** in details that has been discussed in this article. In order to understand the **operating principle of dc motor** we need to first look into its constructional feature.



The very basic construction of a dc motor contains a current carrying armature which is connected to the supply end through commutator segments and brushes and placed within the north south poles of a permanent or an electro-magnet as shown in the diagram below.

For clear understanding the **principle of DC motor** we have to determine the magnitude of the force, by considering the diagram below.

We know that when an infinitely small charge dq is made to flow at a velocity 'v' under the influence of an electric field E, and a magnetic field B, then the Lorentz Force dF experienced by the charge is given by:-



$$dF = dq(E + vB)$$

For the **operation of dc motor**, considering E = 0

$$dF= dq \times v \times B$$

i.e. it's the cross product of dq v and magnetic field B.

$$dF = dq\frac{dL}{dt} \times B \qquad \left[V = \frac{dL}{dt}\right]$$

Where dL is the length of the conductor carrying charge q

$$dF = dq\frac{dL}{dt} \times B$$

$$\text{or, } dF = IdL \times B \qquad \left[\text{Since, current } I = \frac{dq}{dt}\right]$$

$$\text{or. } F = IL \times B = ILB\sin\theta$$

$$\text{or, } F=BIL\sin\theta$$

From the 1st diagram we can see that the construction of a DC motor is such that the direction of current through the armature conductor at all instance is perpendicular to the field. Hence the force acts on the armature conductor in the direction perpendicular to the both uniform field and current is constant.

$$\text{i.e. } \theta = 90°$$

So if we take the current in the left hand side of the armature conductor to be I, and current at right hand side of the armature conductor to be − I, because they are flowing in the opposite direction with respect to each other.
Then the force on the left hand side armature conductor,

$$F_i = BILsin90° = BIL$$

Similarly force on the right hand side conductor

$$F_r = B(-I)Lsin90° = -BIL$$

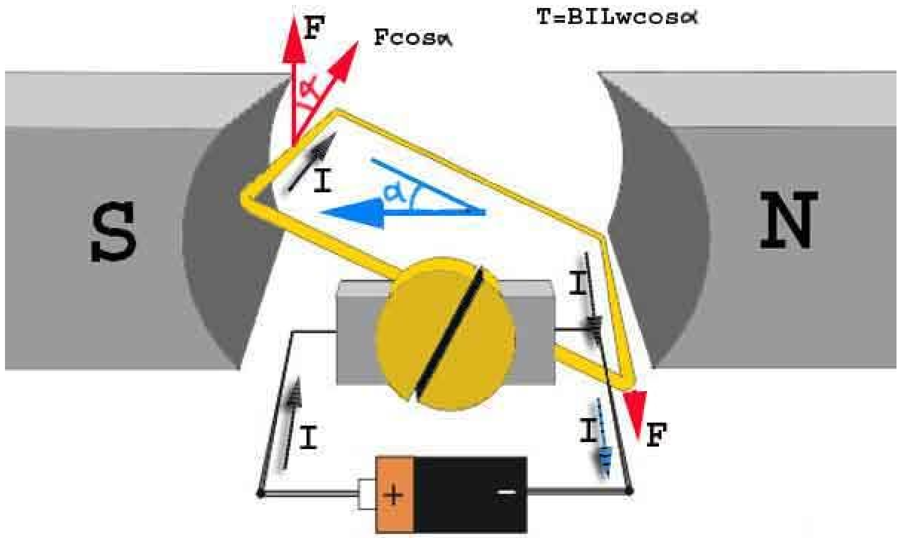
∴ we can see that at that position the force on either side is equal in magnitude but opposite in direction. And since the two conductors are separated by some distance w = width of the armature turn, the two opposite forces produces a rotational force or a torque that results in the rotation of the armature conductor.
Now let's examine the expression of torque when the armature turn crates an angle of α with its initial position.
The torque produced is given by,

Torque = (force, tangential to the direction of armature rotation) × (distance)

Or, $\tau = F\cos\alpha \times \omega$

Or, $\tau = BIL\omega\cos\alpha$

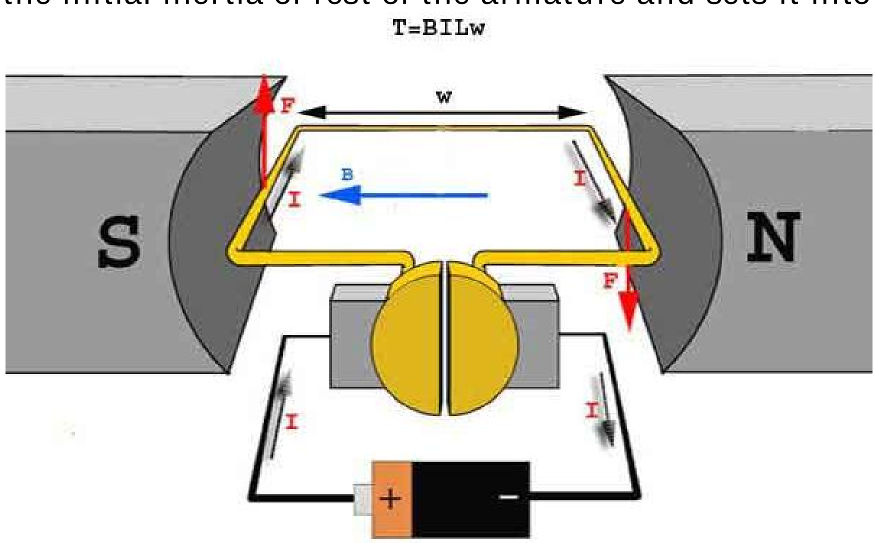
Where α is the angle between the plane of the armature turn and the plane of reference or the initial position of the armature which is here along the direction of magnetic field.
The presence of the term cosα in the torque equation very well signifies that unlike force the torque at all position is not the same. It in fact varies with the variation of the angle α. To explain the variation of torque and the principle behind rotation of the motor let us do a step wise analysis.



Step 1: Initially considering the armature is in its starting point or reference position where the angle α = 0.

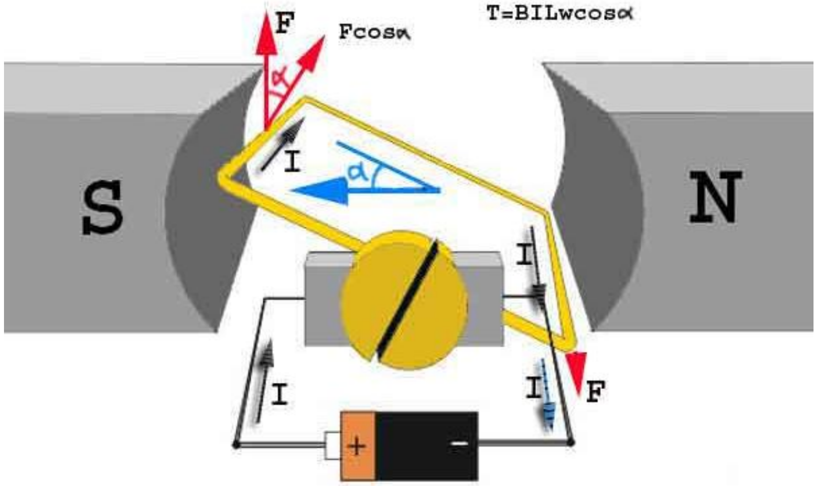
$$\therefore \tau = BIL\omega \times COS\, 0° = BIL\omega$$

Since α = 0, the term cos α = 1, or the maximum value, hence torque at this position is maximum given by $\tau = BIL\omega$. This high starting torque helps in overcoming the initial inertia of rest of the armature and sets it into rotation.



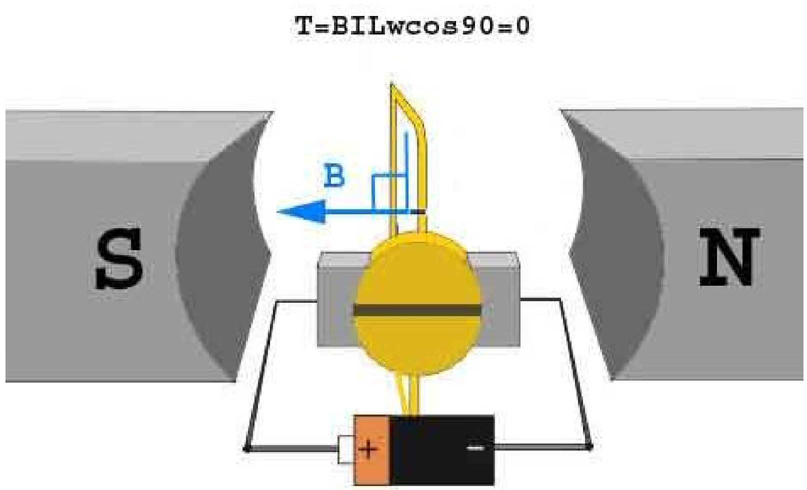
Step 2: Once the armature is set in motion, the angle α between the actual position of the armature and its reference initial position goes on increasing in

the path of its rotation until it becomes 90° from its initial position. Consequently the term cosα decreases and also the value of torque.
The torque in this case is given by $\tau = BIL\omega cos\alpha$ which is less than BIL w when α is greater than 0°.



Step 3: In the path of the rotation of the armature a point is reached where the actual position of the rotor is exactly perpendicular to its initial position, i.e. α = 90°, and as a result the term cosα = 0.
The torque acting on the conductor at this position is given by,

$$\tau = BIL\omega \cos 90° = 0$$



i.e. virtually no rotating torque acts on the armature at this instance. But still the armature does not come to a standstill this is because of the fact that the operation of dc motor has been engineered in such a way that the inertia of motion at this point is just enough to overcome this point of null torque. Once the rotor crosses over this position the angle between the actual position of the armature and the initial plane again decreases and torque starts acting on it again.

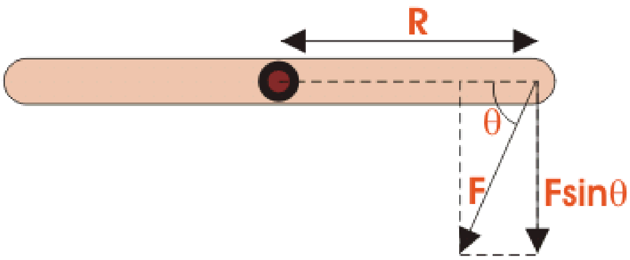
Torque Equation of DC Motor

The term torque as best explained by Dr. Huge d Young is the quantitative measure of the tendency of a force to cause a rotational motion, or to bring about a change in rotational motion. It is in fact the moment of a force that produces or changes a rotational motion.

Where F is force in linear direction.

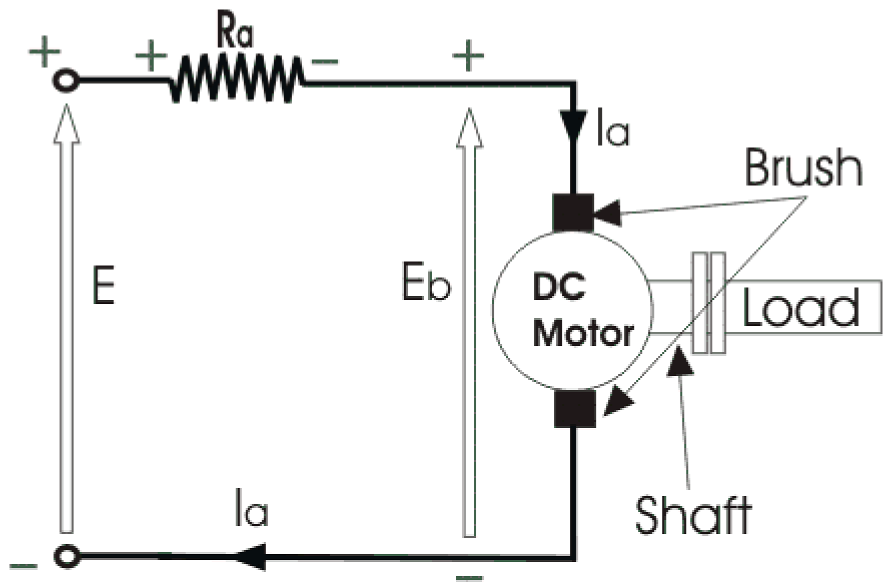
R is radius of the object being rotated,

and θ is the angle, the force F is making with R vector



The dc motor as we all know is a rotational machine, and **torque of dc motor** is a very important parameter in this concern, and it's of utmost importance to understand the **torque equation of dc motor** for establishing its running characteristics.

To establish the torque equation, let us first consider the basic circuit diagram of a dc motor, and its voltage equation.



Referring to the diagram beside, we can see, that if E is the supply voltage, E_b is the back emf produced and I_a, R_a are the armature current and armature resistance respectively then the voltage equation is given by,

$$E = E_b + I_a R_a \quad \ldots\ldots\ldots\ldots\ldots (2)$$

But keeping in mind that our purpose is to derive the **torque equation of dc motor** we multiply both sides of equation (2) by I_a.

Therefore, $$EI_a = E_b I_a + {I_a}^2 R_a \quad \ldots\ldots\ldots\ldots\ldots (3)$$

Now $I_a^2.R_a$ is the power loss due to heating of the armature coil, and the true effective mechanical power that is required to produce the desired torque of dc machine is given by,

$$P_m = E_b I_a \quad \ldots\ldots\ldots (4)$$

The mechanical power P_m is related to the electromagnetic torque T_g as,

$$P_m = T_g \omega \quad \ldots\ldots\ldots (5)$$

Where ω is speed in rad/sec.

Now equating equation (4) & (5) we get,

$$T_g \omega = E_b I_a$$

$$E_b I_a = T_g \omega$$

Now for simplifying the torque equation of dc motor we substitute.

$$E_b = \frac{\phi PNZ}{60A} \text{ .... } (6)$$

Where, P is no of poles,

φ is flux per pole,

Z is no. of conductors,

A is no. of parallel paths,

and N is the speed of the D.C. motor.

$$Hence,\ \boldsymbol{\omega} = \frac{\boldsymbol{2\pi N}}{\boldsymbol{60}} \text{ .... } (7)$$

Substituting equation (6) and (7) in equation (4), we get:

$$T_g = \frac{PZ\phi I_a}{2\pi A}$$

The torque we so obtain, is known as the electromagnetic torque of dc motor, and subtracting the mechanical and rotational losses from it we get the mechanical torque.

Therefore, $T_m = T_g$ - mechanical losses.

This is the torque equation of dc motor. It can be further simplified as:

$$T_g = K_a \phi I_a$$

$$\text{Where, } K_a = \frac{PZ}{2\pi}$$

Which is constant for a particular machine and therefore the torque of dc motor varies with only flux φ and armature current I_a.

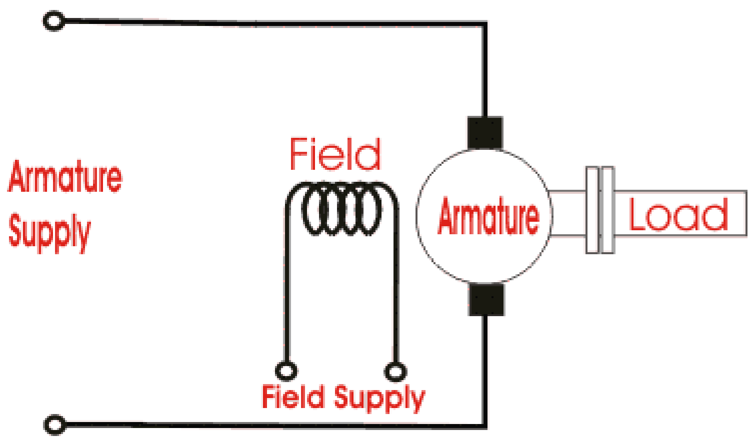
Types of DC Motors

The direct current motor or the DC motor has a lot of application in today's field of engineering and technology. Starting from an electric shaver to parts of automobiles, in all small or medium sized motoring applications DC motors come handy. And because of its wide range of application different functional types of dc motor are available in the market for specific requirements. The types of DC motor can be listed as follows

Separately Excited DC Motor:

As the name suggests, in case of a separately excited DC motor the supply is given separately to the field and armature windings. The main distinguishing fact in these types of dc motor is that, the armature current does not flow through the field windings, as the field winding is energized from a separate external source of dc current as shown in the figure beside.

From the torque equation of dc motor we know $T_g = K_a \varphi I_a$ So the torque in this case can be varied by varying field flux φ, independent of the armature current I_a.



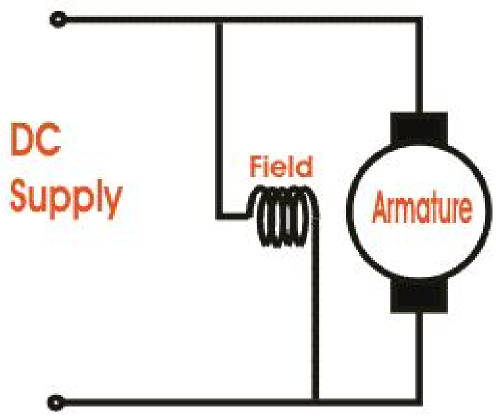
Self Excited DC Motor

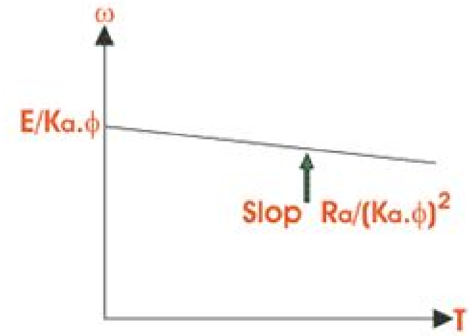
In case of self excited dc motor, the field winding is connected either in series or in parallel or partly in series, partly in parallel to the armature winding, and on this basis its further classified as:-

1. **Shunt wound DC motor**.
2. **Series wound DC motor**.
3. **Compound wound DC motor**.

Shunt Wound DC Motor:

In case of a shunt wound dc motor or more specifically shunt wound self excited dc motor, the field windings are exposed to the entire terminal voltage as they are connected in parallel to the armature winding as shown in the figure below.





To understand the characteristic of these types of DC motor, lets consider the basic voltage equation given by,

$$E = E_b + I_a R_a \ldots\ldots\ldots\ldots (8)$$

[Where E, E_b, I_a, R_a are the supply voltage, back emf, armature current and armature resistance respectively]

$$E_b = K_a \emptyset \omega \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

[since back emf increases with flux φ and angular speed ωω]

Now substituting E_b from equation (9) to equation (8) we get,

$$E = K_a \emptyset \omega + I_a R_a$$

$$\therefore \omega = \frac{E - I_a R_a}{K_a \emptyset} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The torque equation of a dc motor resembles,

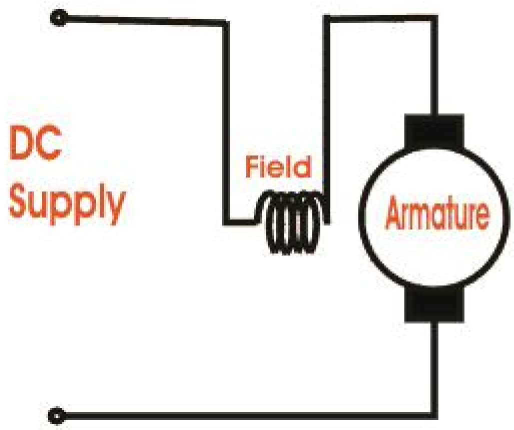
$$T_g = K_a \emptyset I_a \ldots\ldots\ldots\ldots\ldots (11)$$

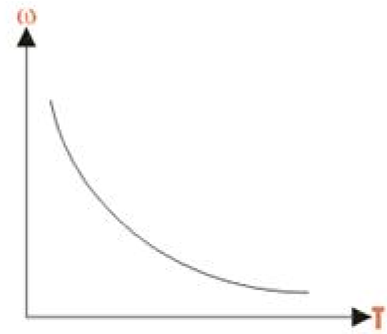
This is similar to the equation of a straight line, and we can graphically representing the torque speed characteristic of a shunt wound self excited dc motor as

The shunt wound dc motor is a constant speed motor, as the speed does not vary here with the variation of mechanical load on the output.

Series Wound DC Motor

In case of a series wound self excited dc motor or simply **series wound dc motor**, the entire armature current flows through the field winding as its connected in series to the armature winding. The series wound self excited dc motor is diagrammatically represented below for clear understanding.





Now to determine the torque speed characteristic of these types of DC motor, let's get to the torque speed equation.

From the circuit diagram we can see that the voltage equation gets modified to

$$E = E_b + I_a(R_a + R_{se})$$

Whereas back emf remains $E_b = K_a \emptyset \omega$

Neglecting saturation we get,

$$\emptyset = K_1 I_f = K_1 I_a$$

[since field current = armature current]

$$\text{Therefore, } E_b = K_a K_1 I_a \omega = K_s I_a \omega$$

$$\omega = \frac{E}{K_s I_a} - \frac{(R_a + R_{se})}{K_s}$$

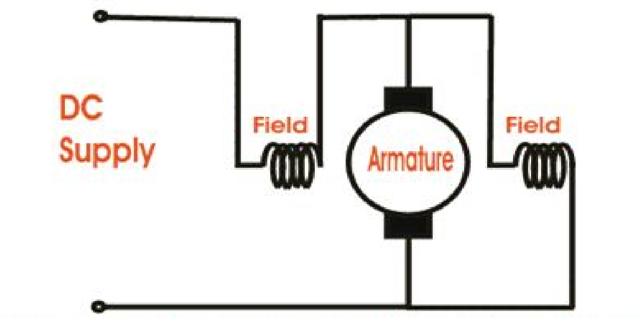
From equations we obtain the torque speed characteristic as shown in above figure

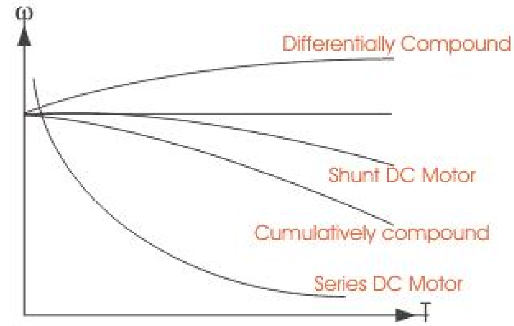
In a series wound dc motor, the speed varies with load. And operation wise this is its main difference from a shunt wound dc motor.

Compound Wound DC Motor

The compound excitation characteristic in a dc motor can be obtained by combining the operational characteristic of both the shunt and series excited dc motor. The compound wound self excited dc motor or simply **compound wound dc motor** essentially contains the field winding connected both in series and in parallel to the armature winding as shown in the figure below:

The excitation of compound wound dc motor can be of two types depending on the nature of compounding.





Cumulative Compound DC Motor

When the shunt field flux assists the main field flux, produced by the main field connected in series to the armature winding then it is called cumulative compound dc motor.

$$\emptyset_{total} = \emptyset_{series} + \emptyset_{shunt}$$

Differential Compound DC Motor

In case of a differentially compounded self excited dc motor i.e. differential compound dc motor, the arrangement of shunt and series winding is such that the field flux produced by the shunt field winding diminishes the effect of flux by the main series field winding.

$$\emptyset_{total} = \emptyset_{series} - \emptyset_{shunt}$$

The net flux produced in this case is lesser than the original flux and hence does not find much of a practical application.

The compounding characteristic of the self excited dc motor is shown in the figure above.

Both the cumulative compound and differential compound dc motor can either be of short shunt or long shunt type depending on the nature of arrangement.

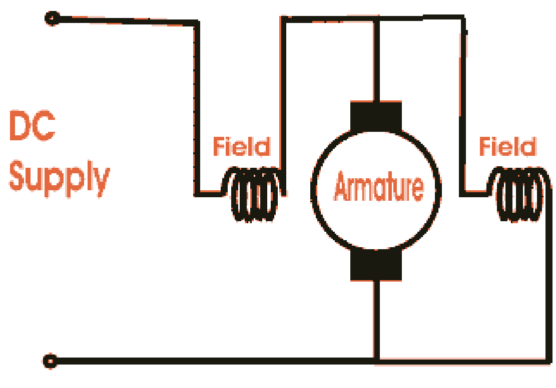
Short Shunt DC Motor

If the shunt field winding is only parallel to the armature winding and not the series field winding then it's known as short shunt dc motor or more specifically short shunt type compound wound dc motor.

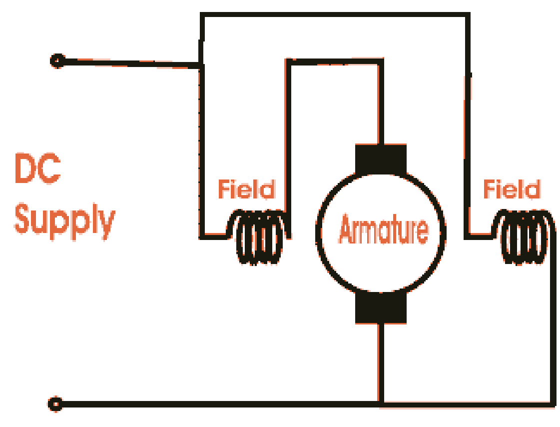
Long Shunt DC Motor

If the shunt field winding is parallel to both the armature winding and the series field winding then it's known as long shunt type compounded wound dc motor or simply long shunt dc motor.

Short shunt and long shunt type motors have been shown in the diagram below.

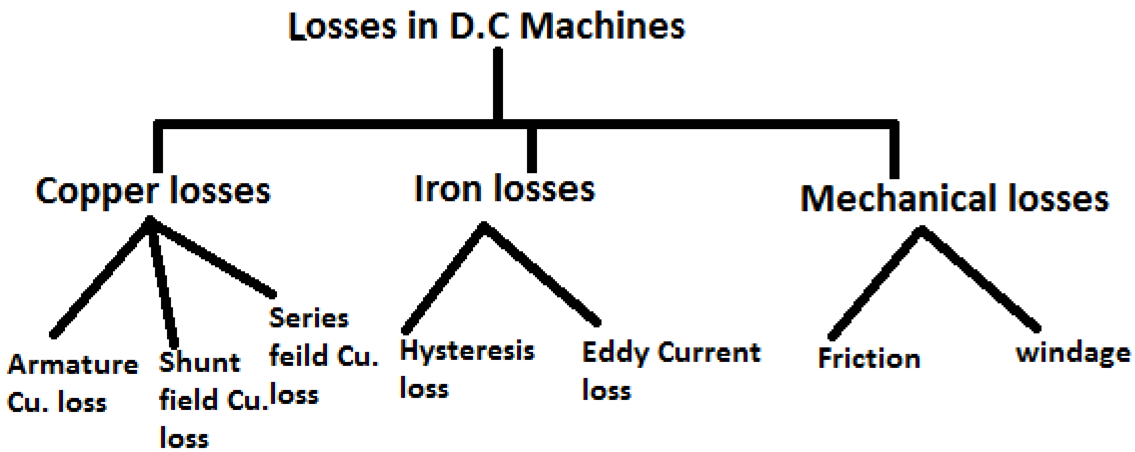


Short Shund DC Motor



Long Shunt DC Motor

Power losses in a DC machine:



The **losses in a dc machine (generator or motor)** may be divided into three classes. They are

(i) Copper losses
(ii) Iron or core losses and
(iii) Mechanical losses.

All these losses appear as heat and thus raise the temperature of the machine. They also lower the efficiency of the machine.

Copper losses

These losses occur due to currents in the various windings of the machine.

1. Armature copper loss = $I_a^2R_a$
2. Shunt field copper loss = $I_{sh}^2R_{sh}$
3. Series field copper loss = $I_{se}^2R_{se}$

Note. There is also brush contact loss due to brush contact resistance (i.e., resistance between the surface of brush and surface of commutator). This loss is generally included in armature copper loss.

Iron or Core losses

These losses occur in the armature of a D.C. machine and are due to the rotation of armature in the magnetic field of the poles. They are of two types viz., (i) hysteresis loss (ii) eddy current loss.

(i) Hysteresis loss

Hysteresis loss occurs in the armature of the d.c. machine since any given part of the armature is subjected to magnetic field reversals as it passes under successive poles.

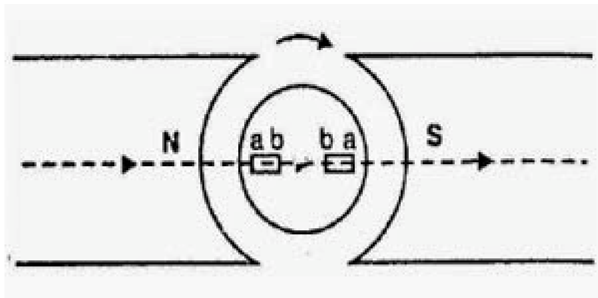


Figure shows an armature rotating in two-pole machine. Consider a small piece ab of the armature. When the piece ab is under N-pole, the magnetic lines pass from a to b. Half a revolution later, the same piece of iron is under S-pole and magnetic lines pass from b to a so that magnetism in the iron is reversed. In order to reverse continuously the molecular magnets in the armature core, some amount of power has to be spent which is called hysteresis loss. It is given by Steinmetz formula. This formula is

$$\textbf{Hysteresis loss } P_h = \eta B_{max}^{1.6} f V$$

Where Bmax = Maximum flux density in armature

f = Frequency of magnetic reversals

= NP/120 where N is in r.p.m

V = Volume of armature in m3

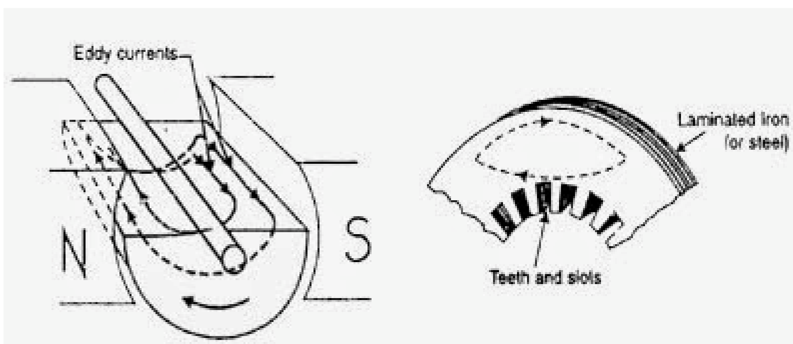
h = Steinmetz hysteresis co-efficient

In order to reduce this loss in a d.c. machine, armature core is made of such materials which have a low value of Steinmetz hysteresis co-efficient e.g. silicon steel.

(ii) Eddy current loss

In addition to the voltages induced in the armature conductors, there are also voltages induced in the armature core. These voltages produce circulating currents in the armature core as shown in Fig. These are called eddy currents and power loss due to their flow is called eddy current loss. The eddy current loss appears as heat which raises the temperature of the machine and lowers its efficiency.

If a continuous solid iron core is used, the resistance to eddy current path will be small due to large cross-sectional area of the core. Consequently, the magnitude of eddy current and hence eddy current loss will be large. The magnitude of eddy current can be reduced by making core resistance as high as practical. The core resistance can be greatly increased by constructing the core of thin, round iron sheets called laminations. The laminations are insulated from each other with a coating of varnish. The insulating coating has a high resistance, so very little current flows from one lamination to the other. Also, because each lamination is very thin, the resistance to current flowing through the width of a lamination is also quite large. Thus laminating a core increases the core resistance which decreases the eddy current and hence the eddy current loss.



Eddy current loss $P_e = K_e B_{max} f^2 t^2 V$

Where Ke = Constant depending upon the electrical resistance of core and system of units used

Bmax = Maximum flux density in Wb/m_2
f = Frequency of magnetic reversals in Hz
t = Thickness of lamination in m
V = Volume of core in m_3

It may be noted that eddy current loss depends upon the square of lamination thickness. For this reason, lamination thickness should be kept as small as possible.

Mechanical losses

These losses are due to friction and windage.
(i) Friction loss e.g., bearing friction, brush friction etc.
(ii) windage loss i.e., air friction of rotating armature.
These losses depend upon the speed of the machine. But for a given speed, they are practically constant.
Note. Iron losses and mechanical losses together are called **stray losses**.

Constant and Variable Losses

The losses in a d.c. generator (or d.c. motor) may be sub-divided into (i) constant losses (ii) variable losses.

(i) Constant losses Those losses in a d.c. generator which remain constant at all loads are known as constant losses. The constant losses in a d.c. generator are:

(a) iron losses
(b) mechanical losses
(c) shunt field losses

(ii) Variable losses
Those losses in a d.c. generator which vary with load are called variable losses. The variable losses in a d.c. generator are:

(a) Copper loss in armature winding ($I^2_a R_a$)
(b) Copper loss in series field winding ($I_{se}^2 R_{se}$)

Total losses = Constant losses + Variable losses

Condition for Maximum Efficiency:

The efficiency of a d.c. generator is not constant but varies with load. Consider a shunt generator delivering a load current I_L at a terminal voltage V.

Generator output = V I_L

Generator input = Output + Losses

= V I_L + Variable losses + Constant losses

= $VI_L + I_a^2 R_a + W_C$

$$= VI_L + (I_L + I_{sh})^2 R_a + W_c \qquad (since\ I_a = I_L + I_{sh})$$

The shunt field current I_{sh} is generally small as compared to I_L and therefore, it can be neglected.

Generator input = $VI_L + I^2_L R_a + W_C$

$$\eta = \frac{output}{input} = \frac{VI_L}{VI_L + I_L{}^2 R_a + W_c}$$

$$= \frac{1}{1 + \left[\frac{I_L R_a}{V} + \frac{W_c}{VI_L}\right]}$$

The efficiency will be maximum when the denominator of above Equation is minimum i.e.,

$$\frac{d}{dI_L}\left[\frac{I_L R_a}{V} + \frac{W_c}{VI_L}\right] = 0$$

$$\frac{R_a}{V} + \frac{W_c}{VI_L{}^2} = 0$$

$$or\ \frac{R_a}{V} = \frac{W_c}{VI_L{}^2}$$

$$or\ I_L{}^2 R_a = W_c$$

Hence Variable loss = Constant loss

The load current corresponding to maximum efficiency is given by;

$$I_L = \sqrt{\frac{W_c}{R_a}}$$

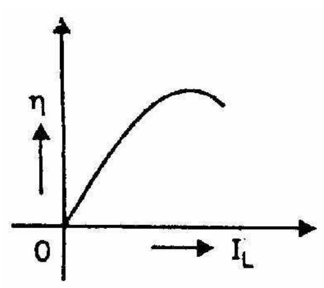


Fig 2.22 Efficiency curve of a DC generator

APPLICATIONS OF DC Motors:

D.C. Series Motor. Since it has high starting torque and variable speed, it is used for heavy duty applications such as electric locomotives, steel rolling mills, hoists, lifts and cranes.

D.C. Shunt Motor. It has medium starting torque and a nearly constant speed. Hence, it is used for driving constant-speed line shafts, lathes, vacuum cleaners, wood-working machines, laundry washing machines, elevators, conveyors, grinders and small printing presses etc.

Cumulative Compound Motor. It is a varying-speed motor with high starting torque and is used for driving compressors, variable-head centrifugal pumps, rotary presses, circular saws, shearing machines, elevators and continuous conveyors etc.

UNIT IV
Assignment Cum Tutorial

Section - A

Objective Questions

1) The armature of a DC machine is laminated to reduce [ ]

a) Eddy current Loss b) Hysteresis loss

c) Copper loss d) Friction and windage loss

2) The direction of induced emf in motor is given by [ ]

a) Fleming Right hand rule b) Fleming Left hand rule

c) Fleming Right hand screw rule d) none

3) HP rating on name plate of a DC motor indicates [ ]

a) output power b) input power c) both d) any of these

4) The emf generated in a DC motor is called back emf because [ ]

a)it is generated in the armature

b) it opposes the direction of rotation of the motor

c) it is in a direction opposite to the applied voltage

d)none of these

5) The armature core of a DC machine is made up of [ ]

a) solid aluminium b) laminated aluminium

c) solid steel d) laminated steel.

6) Rotating part of DC motor is known as [ ]

a) Pole b) Armature c) Carbon brush d) Starter

7) Direction of rotation of DC motor is reversed by [ ]

a) Reversing supply connection.
b) Adding resistance to field circuit.
c) Interchanging armature and field connection.
d) Reversing armature connection or field connection.

8) DC series motor should start with [ ]

a) No load b) Full load c) Half load d) 70.7% load

9) Which of the following represents the torque developed by a DC motor?

a) Torque α I_a b) Torque α Ø c) Torque α I_a Ø d) none of these

10) Which of the following is a cause for the production of back emf?
a) generator action b) motor action [ ]
c) armature reaction d) none of these.
11) Which of the following is the cause for the production of torque in a dc motor? [ ]
a) the resultant flux due to the field and armature currents
b)the flux due to field current
c)the flux due to armature current
d) none of these
12) In a DC motor unidirectional torque is produced with the help of[ ]
a) brushes b) commutator c) end-plates d) both a and b
13) The current drawn by the a 230 V DC motor of armature resistance 0.5 Ω and back emf 200 V is [ ]
a) 60 b) 40 c) 600 d) 660

Section – B

Descriptive Questions

1) Explain briefly the construction and operation of a DC machine.
2) List and explain different types of DC motors. Also mention their applications.
3) Derive an expression for the torque developed by a DC motor.
4) Explain various losses that occur in a DC machine.
5) A 220V DC shunt machine has an armature resistance of 0.5Ω.if the full load armature current is 20A.. Find the induced emf when the machine acts as motor?
6) A DC Motor takes an armature current of 110A at 480V. The armature circuit resistance is 0.2Ω. The machine has 6 poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05wb. Calculate i) speed and ii) the gross torque developed by the armature.

7) A 250V, 4 pole, wave wound d.c series motor has 782 conductors on its

armature. It has armature and series field resistance of 0.75Ω. The motor takes a current of 40A. Estimate its speed and gross torque developed if it has a flux per pole of 25mwb.

8) A 250V, 4 pole wave wound DC shunt motor has armature with 500 conductors. The armature resistance is 0.22, field resistance is 150 Ω and flux per pole is 0.02wb. Determine the speed and torque developed, if the motor draws 16A from the mains.

9) A 400V, 8 pole, DC shunt motor has a wave connected armature winding with 1000 conductors. The useful flux per pole is 0.015wb and the armature and field resistances are 0.4 Ω and 200 Ω respectively. Ignoring the effect of armature reaction, find the speed and the total developed torque when a current of 25A is taken from the mains. If the iron, friction, and windage losses to aggregate to 1000W, find the useful torque, brake horse power, and efficiency at this speed.

10) A 4 pole DC motor takes a 50A armature current. The armature has lap connected 480 conductors. The flux per pole is 20mwb. Calculate the gross torque developed by the armature of the motor.

11) A 4 pole, 250V, DC series motor has a wave connected armature with 200 conductors. The flux per pole is 25mwb when the motor is drawing 60A from the supply. Armature resistance is 0.15Ω while series field winding resistance is 0.2 Ω. Calculate the speed under this condition.

12) A 6 pole, 500V wave connected shunt motor has 1200 armature conductors and useful flux/pole of 20mwb. The armature and field resistance are 0.5 Ω and 250 Ω respectively. What will be the speed and torque developed by the motor when it draws 20A from the supply mains? Neglect armature reaction. If magnetic and mechanical losses amount to 900W, find i) useful torque ii) output in KW and iii) efficiency at this load.

Section- C

GATE / IES Questions

1) A 250v,D.C,shunt motor has shunt field Resistance of 250Ω and an armature resistance of 0.25Ω. for a given load torque and no additional resistance is included in the shunt field circuit, the motor runs at 1500rpm.and drawing an armature current of 20A. If a resistance of 250Ω is inserted in series with the field circuit, the load torque remains same, find the new speed and armature current. **(GATE 2010)**

2) A 220-V,10Kw, 2500r.p.m. shunt motor draws 41A when operating at rated conditions. The resistance of the armature and shunt field winding are 0.35Ω and 110Ω . calculate the values of armature current and motor speed if pole flux is reduced by 25% , a 1Ω Resistance is placed in series with the armature and load torque is reduced by 50% **(GATE 2013)**

UNIT V
TRANSFORMERS

Objectives:

- To impart some basic knowledge on transformers to students.
- To familiarize students with the applications of transformers.

Syllabus:

Principles of operation, Constructional details, transformer losses, OC and SC tests, Efficiency and Regulation Calculations. (All the above topics are only elementary treatment and simple problems).

Learning Outcomes:

Student will be able to

- Understand the principle of operation of transformer.
- Understand the construction of transformer.
- Determine the efficiency and regulation.

Introduction

The main advantage of alternating currents over direct current is that, the alternating currents can be easily transferable from low voltage to high voltage or high voltage to low. Alternating voltages can be raised or lowered as per requirements in the different stages of electrical network as generation, transmission, distribution and utilization. This is possible with a static device called transformer. The transformer works on the principle of mutual induction. It transfer an electric energy from one circuit to other when there is no electrical connection between the tow circuits. Thus we can define transformer as below :

Key point : The transformer is a static piece of apparatus by means of which an electrical power is transformed from one alternating current circuit to another with the desired change in voltage and current, without any change in the frequency.

The use of transformers in transmission system is shown in the Fig 1.1.

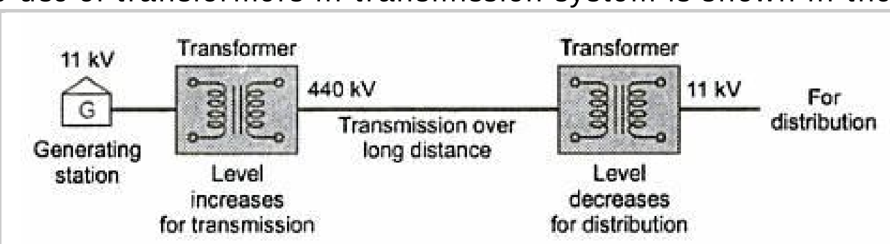


Fig. 1.1 Use of transformer in transmission system

Principle of operation

The principle of mutual induction states that when tow coils are inductively coupled and if current in one coil is changed uniformly then an e.m.f. gets induced in the other coil. This e.m.f can drive a current, when a closed path is provided to it. The transformer works on the same principle. In its elementary form, it consists of tow inductive coils which are electrically separated but linked through a common magnetic circuit. The two coils have high mutual inductance. The basic transformer is shown in the Fig 1.2.

One of the two coils is connected to source of alternating voltage. This coil in which electrical energy is fed with the help of source called primary winding (P). The other winding is connected to load. The electrical energy transformed to this winding is drawn out to the load.

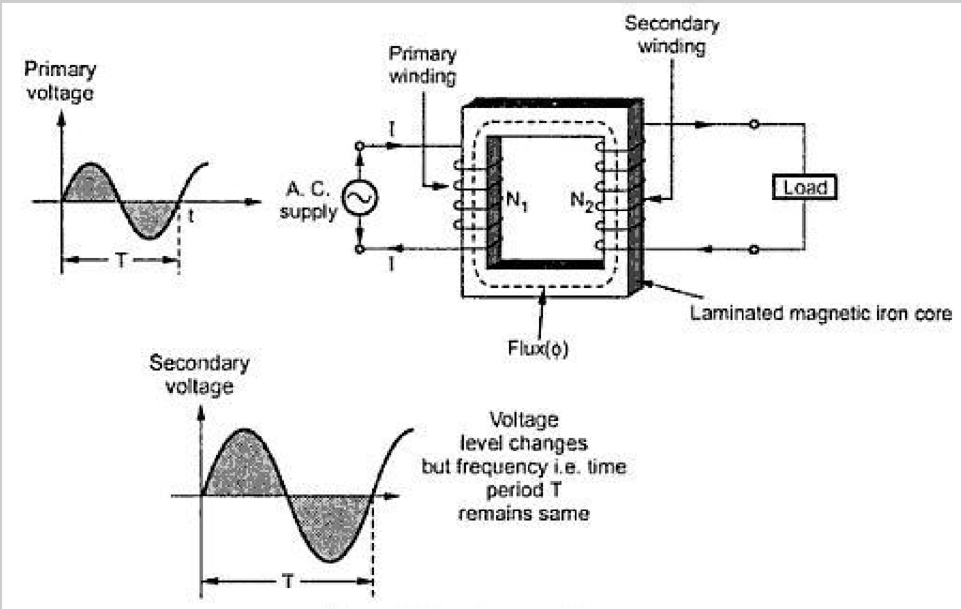


Fig.1.2 Basic transformer

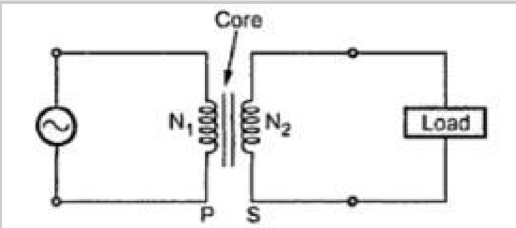


Fig 1.3 Symbolic representation

This winding is called secondary winding (S). The primary winding has N_1number of turns while the secondary winding has N_2 number of turns. Symbolically the transformer is indicated as shown in the Fig 1.3.

When primary winding is excited by an alternating voltage, it circulates an alternating current. This current produces an alternating flux (Φ)which completes its path through common magnetic core as shown dotted in the Fig 1.2. Thus an alternating, flux links with the secondary winding. As the flux is alternating, according to Faraday's law of an electromagnetic induction, mutually induced e.m.f. gets developed in the secondary winding. If now load is connected to the secondary winding, this e.m.f. drives a current through it.

Thus through there is no electrical contact between the two windings, an electrical energy gets transferred from primary to the secondary.

Key point : The frequency of the mutual induced e.m.f. is same as that of the alternating source which is supplying energy to the primary winding.

Can D.C. Supply be used for Transformer?

The d.c. supply can not be used for the transformers.

The transformer works on the principle of mutual induction, for which current in one coil must change uniformly. If d.c. supply is given, the current will not change due to constant supply and transformer will not work.
Practically winding resistance is very small. For d.c., the inductive reactance X_L is zero as d.c. has no frequency. So total impedance of winding is very low for d.c. Thus winding will draw very high current if d.c. supply is given to it. This may cause the burning of windings due to extra heat generated and may cause permanent damage to the transformer.

There can be saturation of the core due to which transformer draws very large current from the supply when connected to d.c.

Thus d.c. supply should not be connected to the transformers.

Construction of Transformer

There are two basic parts of a transformer i) Magnetic Core ii) Winding or Coils.

The core of the transformer is either square or rectangular in size. It is further divided into tow parts. The vertical position on which coils are wound is called limb while the top and bottom horizontal portion is called yoke of the core. These parts are shown in the Fig.1(a).

Core is made up of lamination. Because of laminated type of construction, eddy current losses get minimised. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air gap at joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the Fig 1(b).

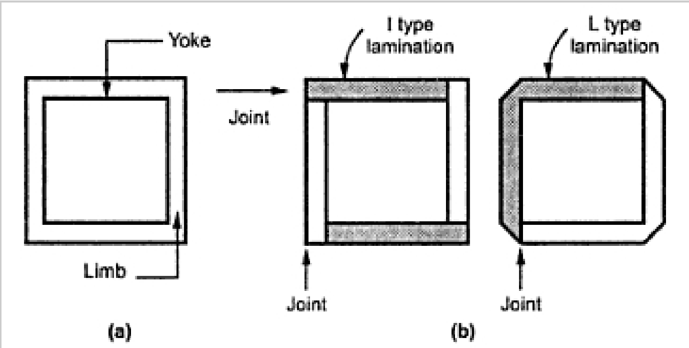


Fig. 1 Construction of transformer

The cross-section of the limb depends on the type of coil to be used either circular or rectangular. The different cross-section of limbs, practically used are shown in the Fig. 2.

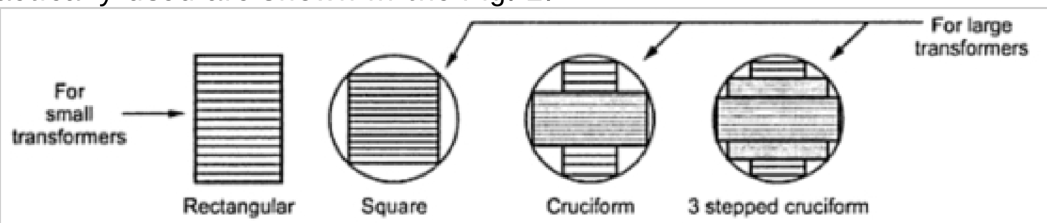


Fig. 2 Different cross-sections

Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Fig 1.2 (see post : Working Principle of 1-Phase Transformer) the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which effects the transformer performance badly. Similarly it is necessary that the windings should be very closes to each other to have high mutual inductance. To achieve this, the tow windings are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical coils as shown in the Fig. 3.

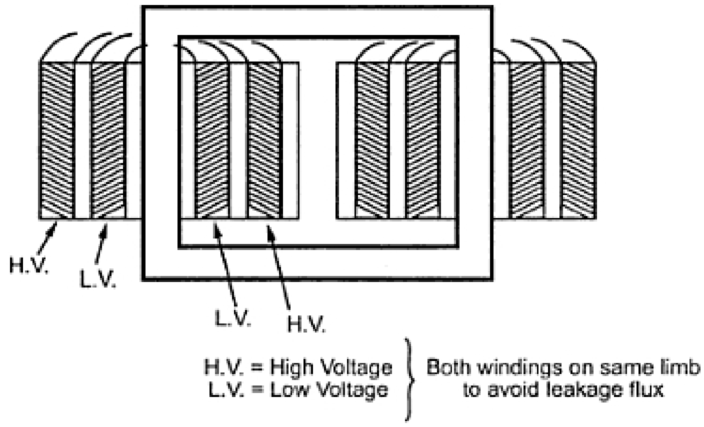


Fig. 3 Cylindrical concentric coils

Such cylindrical coils are used in the core type transformer. Theses coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

The other type of coils which is very commonly used for the shell type of transformer is sandwiching coils. Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portion reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The sandwich coil is shown in the Fig. 4. The top and bottom coils are low voltage coils. All the portion are insulated from each other by paper.

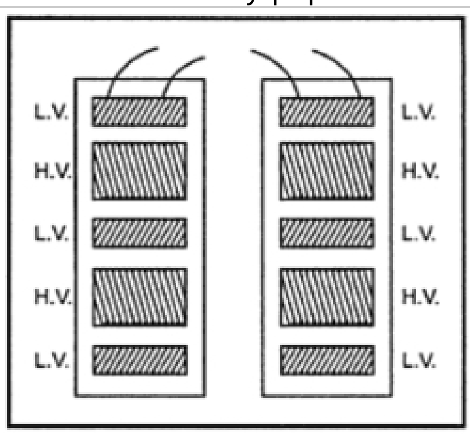


Fig. 4 Sandwich coils

The various constructions used for the single phase transformers are,
1. Core type 2. shell type and 3. Berry type

1. Core Type Transformer

It has a single magnetic circuit. The core rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil. Core is made up of large number of thin laminations.

As The windings are uniformly distributed over the two limbs, the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

The Fig. 1(a) shows the schematic representation of the core type transformer while the Fig 1(b) shows the view of actual construction of the core type transformer.

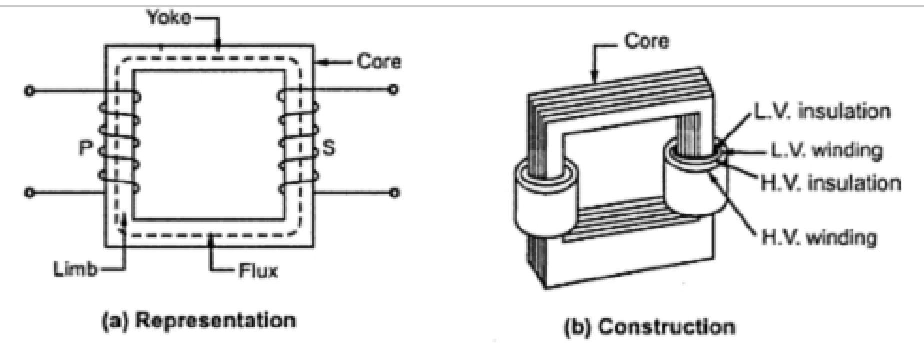


Fig. 1 Core type transformer

2. Shell Type Transformer

It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between tow low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called over lapped or imbricated joint. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

The Fig. 2(a) shows the schematic representation while the Fig. 2(b) shows the outaway view of the construction of the shell type transformer.

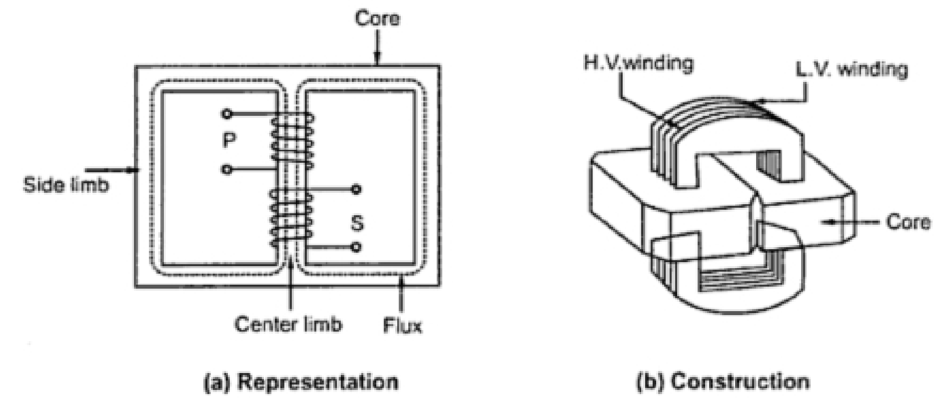


Fig 2 Shell type transformer

3. Berry Type Transformer

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

Diagrammatically it can be shown as in the Fug. 3.

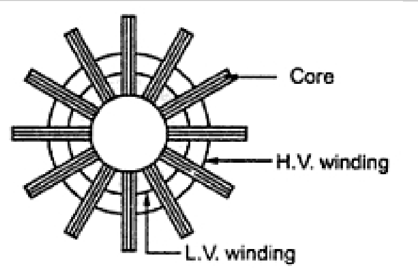


Fig. 3 Berry type transformer

The transformers are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into the rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutes leaks. The tanks are filled with the special insulating oil. The entire transformer assembly is immersed in the oil. Oil serves two functions : i) Keeps the coil cool by circulation and ii) Provides the transformers an additional insulation.

The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are sealed air tight to avoid the contact of oil with atmospheric air and moisture. In large transformers, the chambers called breather are provided. The breathers prevent the atmospheric moisture to pass on to the oil. The breathers contain the silica gel crystal which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge is formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called conservator.

E.M.F Equation of a Transformer

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux Φ. The primary winding has N_1number of turns. The alternating flux Φ linking with the primary winding itself induces an e.m.f in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of Φ_m as show in the Fig. 1.

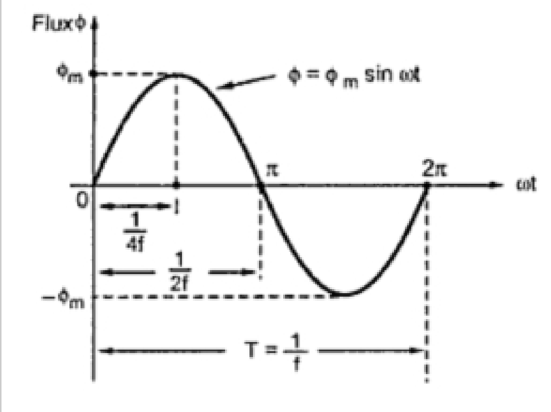


Fig. 1 Sinusoidal flux

The various quantities which affect the magnitude of the induced e.m.f. are :

Φ = Flux
Φ_m = Maximum value of flux
N_1 = Number of primary winding turns
N_2 = Number of secondary winding turns
f = Frequency of the supply voltage
E_1 = R.M.S. value of the primary induced e.m.f.
E_2 = R.M.S. value of the secondary induced e.m.f.

From Faraday's law of electromagnetic induction the voltage e.m.f. induced in each turn is proportional to the average rate of change of flux.

∴ average e.m.f. per turn = average rate of change of flux

∴ average e.m.f. per turn = $d\Phi/dt$

Now $d\Phi/dt$ = Change in flux/Time required for change in flux

Consider the 1/4 th cycle of the flux as shown in the Fig.1. Complete cycle gets completed in 1/f seconds. In 1/4 th time period, the change in flux is from 0 to Φ_m.

∴ $d\Phi/dt = (\Phi_m - 0)/(1/4f)$ as dt for 1/4 th time period is 1/4f seconds

$= 4 f \Phi_m$ Wb/sec

∴ Average e.m.f. per turn = $4 f \Phi_m$ volts

As is sinusoidal, the induced e.m.f. in each turn of both the windings is also sinusoidal in nature. For sinusoidal quantity,

From factor = R.M.S. value/Average value = 1.11

∴ R.M.S. value of induced e.m.f. per turn

$= 1.11 \times 4 f \Phi_m = 4.44 f \Phi_m$

There are number of primary turns hence the R.M.S value of induced e.m.f. of primary denoted as is E_1,

$E_1 = N_1 \times 4.44 f \Phi_m$ volts

While as there are number of secondary turns the R.M.S values of induced e.m.f. of secondary denoted is E_2 is,

$E_2 = N_2 \times 4.44 f \Phi_m$ volts

The expression of E_1 and E_2 are called e.m.f. equation of a transformer.

Thus e.m.f. equations are,

$E_1 = 4.44 f \Phi_m N_1$ volts ....(1)

$E_2 = 4.44 f \Phi_m N_2$ volts ....(2)

Ratios of transformer

Consider a transformer shown in Fig.1 indicating various voltages and currents.

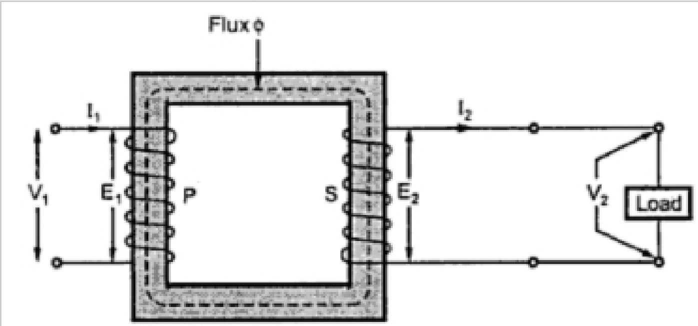


Fig. 1 Ratios of transformer

Voltage Ratio

We known from the e.m.f. equations of a transformer that

$$E_1 = 4.44\ f\ \Phi_m\ N_1 \quad \text{and} \quad E_2 = 4.44\ f\ \Phi_m\ N_2$$

Taking ratio of the two equations we get,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

This ratio of secondary induced e.m.f. to primary induced e.m.f. is known as voltage transformation ratio denoted as K,

Thus,

$$E_2 = K\ E_1 \qquad \text{where} \quad K = \frac{N_2}{N_1}$$

1. If $N_2 > N_1$ i.e. $K > 1$, $E_2 > E_1$ we get then the transformer is called step-up transformer.
2. If $N_2 < N_1$ i.e. $K < 1$, we get $E_2 < E_1$ then the transformer is called step-down transformer.
3. If = i.e. K= 1, we get $E_2 = E_1$ then the transformer is called isolation transformer or 1:1 transformer.

Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties :
i) It has no losses.
ii) Its windings have zero resistance.
iii) Leakage flux is zero i.e. 100% flux produced by primary links with the secondary.
iv) Permeability of core is so high that negligible current is required to establish the flux in it.

Key point : For an ideal transformer, the primary applied voltage V_1 is same as the primary induced e.m.f. V_2 as there are no voltage drops.

Similarly the secondary induced e.m.f. E_2 is also same as the terminal voltage V_2 across the load. Hence for an ideal transformer we can write,

$$\frac{E_2}{E_1} = \frac{V_2}{V_1} = K$$

No transformer is ideal in practice but the value of E_1 is almost equal to V_1 for properly designed transformer.

Current ratio

For an ideal transformer there are no losses. Hence the product of primary voltage V_1 and primary current I_1, is same as the product of secondary voltage V_2 and the secondary current I_2.

So $V_1 I_1$ = input VA and $V_2 I_2$ = output VA

For an ideal transformer,

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Key point : Hence the currents are in the inverse ratio of the voltage transformation ratio.

Voltage ampere rating

When electrical power is transferred from primary winding to secondary there are few power losses in between. These power losses appear in the form of heat which increase the temperature of the device.Now this temperature must be maintained below certain limiting values as it is always harmful from insulation point of view. As current is the main cause in producing heat, the output maximum rating is generally specified as the product of output voltage and output current i.e.$V_2 I_2$. This always indicates that when transformer is operated under this specified rating, its temperature rise will not be excessive. The copper loss (I^2R) in the transformer depends on the current 'I' through the winding while the iron or core loss depends on the voltage 'V' as frequency of operation is constant. None of these losses depend on the power factor (cos Φ) of the load. Hence losses decide the temperature and hence the rating of the transformer. As losses depend on V and I only, the rating of the transformer is specified as a product of these two parameters VxI.

Key point : Thus the transformer rating is specified as the product of voltage and current called VA rating.

On both sides, primary and secondary VA rating remains same. This rating is generally expresses in KVA (kilo volt amperes rating).

Now $V_1 / V_2 = I_2 / I_1 = K$

∴ $V_1 I_1 = V_2 I_2$

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

If V_1 and V_2 are the terminal voltages of primary and secondary then from specified KVA rating we can decide full load currents of primary and secondary, I_1 and I_2. This is the safe maximum current limit which may carry, keeping temperature rise below its limiting value.

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \quad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Key point : The full load primary and secondary currents indicate the safe maximum values of currents which transformer windings can carry.

Example 1 : A single phase, 50 Hz transformer has 80 turns on the primary winding and 400 turns on the secondary winding. The net cross-sectional area of the core is 200 cm^2. If the primary winding is connected at a 240 V , 50 Hz supply, determine :

i) The e.m.f. induced in the secondary winding.

ii) The maximum value of the flux density in the core.

Solution

$N_1 = 80$, $f = 50$ Hz , $N_2 = 400$, $a = 200 \text{ cm}^2 = 200 \times 10^{-4} \text{ cm}^2$

$E_1 = 240$

$K = N_2 / N_1 = 400/80 = 5/1$

∴ $K = E_2 / E_1 = E_2 / 240 = 5/1$

$E_2 = 5 \times 240 = 1200$ V

Now $E_1 = 4.44\ f\ \Phi_m\ N_1$

$240 = 4.44 \times 50 \times \Phi_m \times 80$

∴ $\Phi_m = 240/(4.44 \times 50 \times 80) = 0.01351$ Wb

∴ $B_m = \Phi_m / a = 0.01351/(200 \times 10^{-4}) = 0.6756\ Wb/m^2$

Example 2 : For a single phase transformer having primary and secondary turns of 440 and 880 respectively, determine the transformer KVA rating if half load secondary current is 7.5 A and maximum value of core flux is 2.25 Wb.

Solution

$N_1 = 440$, $N_2 = 880$, $(I_2)_{H.L.} = 7.5$ A,

$f_m = 2.25$ mWb , $E_2 = 4.44\ \Phi_m\ f\ N_2$

Assuming f = 50 Hz,

∴ $E_2 = 4.44 \times 2.25 \times 10^{-3} \times 50 \times 880 = 439.56$ V

$(I_2)_{F.L.}$ = KVA rating / E_2

And $(I_2)_{H.L.} = 0.5\ (I_2)_{F.L.}$

∴ $(I_2)_{H.L.} = 0.5 \times$ (KVA rating / E_2)

∴ 7.5 = 0.5 x (KVA rating / 439.56)

∴ KVA rating $= 2 \times 7.5 \times 439.56 \times 10^{-3}$

= 6.5934 KVA .....(10^{-3} for KVA)

Example 3 : A single phase transformer has 350 primary and 1050 secondary turns. The primary is connected to 400 V, 50 Hz a.c. supply. If the net cross-sectional area of the core is 50 cm^2, calculate i) The maximum value of the flux density in the core ii) The induced e.m.f. in the secondary winding.

Solution

The given value are,

$N_1 = 350$ turns, $N_2 = 1050$ turns

$V_1 = 400$ V , $A = 50\ cm^2 = 50 \times 10^{-4}\ m^2$

The e.m.f. of the transformer is,

$E_1 = 4.44\ f\ \Phi_m\ N_1$

$E_1 = 4.44\ B_m\ A\ f\ N_1$ as $\Phi_m = B_m\ A$

Flux density $B_m = E_1 / (4.44\ A\ f\ N_1)$

$= 400 / (4.44 \times 50 \times 10^{-4} \times 50 \times 350)$ assume $E_1 = V_1$

$= 1.0296\ Wb/m^2$

$K = N_2 / N_1 = 1050/350 = 3$

And $K = E_2 / E_1 = 3$

∴ $E_2 = 3 \times E_1 = 3 \times 400 = 1200$ V

Ideal Transformer on No Load

Consider an ideal transformer on no load as shown in the Fig. 3. The supply voltage is and as it is V_1 an no load the secondary current $I_2 = 0$.

The primary draws a current I_1 which is just necessary to produce flux in the core. As it magnetising the core, it is called magnetising current denoted as I_m. As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature. The magnetising current is I_m is very small and lags V_1 by 30^o as the winding is purely inductive. This I_m produces an alternating flux Φ which is in phase with I_m.

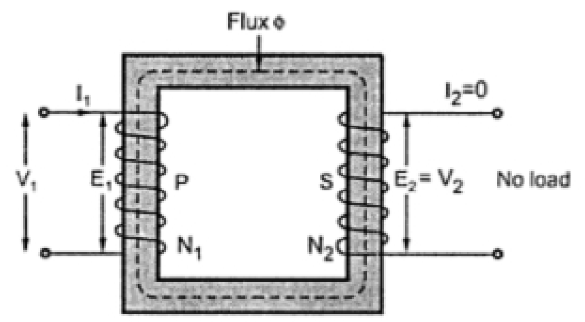


Fig. 1 Ideal transformer on no load

The flux links with both the winding producing the induced e.m.f.s E_1 and E_2 , in the primary and secondary windings respectively. According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence $E_1$is in antiphase with V_1 but equal in magnitude. The induced E_2 also opposes V_1hence in antiphase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.

The phasor diagram for the ideal transformer on no load is shown in the Fig. .2.

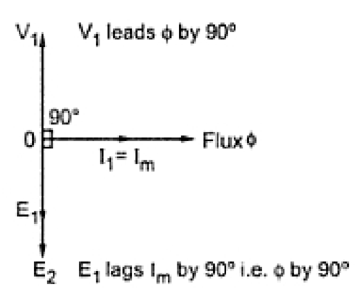


Fig. 2 Phasor diagram for ideal transformer on no load

It can be seen that flux Φ is reference. I_m produces Φ hence in phase with Φ. V_1leads I_m by 90^o as winding is purely inductive so current has to lag voltage by 90^o.

E_1 and E_2 are in phase and both opposing supply voltage .

The power input to the transformer is V_1 I_1 cos (V_1 ^ I_1) i.e. V_1 I_m cos(90^o) i.e. zero. This is because on no load output power is zero and for ideal transformer there are no losses hence input power is also zero. Ideal no load p.f. of transformer is zero lagging.

Practicle Transformer on No Load

Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux. While designing the transformer the efforts are made to keep these losses minimum by,

1. Using high grade material as silicon steel to reduce hysteresis loss.
2. Manufacturing core in the form of laminations or stacks of thin lamination to reduce eddy current loss.

Apart from this there are iron losses in the practical transformer. Practically primary winding has certain resistance hence there are small primary copper loss present.

Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I_o.

Now the no load input current I_o has two components :

1. A purely reactive component I_m called magnetising component of no load current required to produce the flux. This is also called wattless component.
2. An active component I_c which supplies total losses under no load condition called power component of no load current. This also called wattful component or core loss component of I_o.

Th total no load current I_o is the vector addition of I_m and I_c.

$$\bar{I}_o = \bar{I}_m + \bar{I}_c \quad \text{... (1)}$$

In practical transformer, due to winding resistance, no load current I_o is no longer at 90^o with respect to V_1. But it lags V_1 by angle Φ_o which is less than 90^o. Thus $\cos \Phi_o$ is called no load power factor of practical transformer.

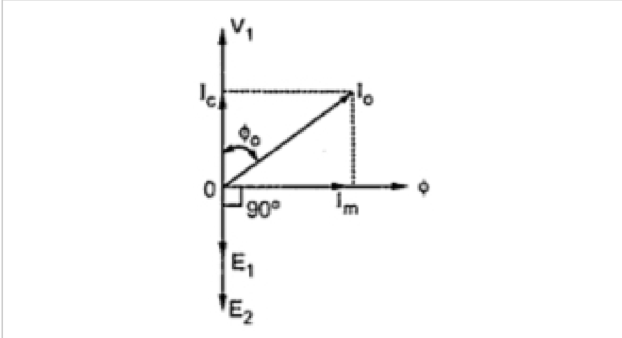


Fig 1. Practical transformer on no load

The phasor diagram is shown in the Fig. 1. It can be seen that the two components I_o are,

$$I_m = I_o \sin \phi_o \quad \text{... (2)}$$

This is magnetising component lagging V_1 exactly by 90^o .

$$I_c = I_o \cos \phi_o \quad \text{... (3)}$$

This is core loss component which is in phase with V_1.
The magnitude of the no load current is given by,

$$I_o = \sqrt{I_m^2 + I_c^2} \quad \text{... (4)}$$

While Φ_o = no load primary power factor angle
The total power input on no load is denoted as W_o and is given by,

$$W_o = V_1 I_o \cos\phi_o = V_1 I_c \quad \text{... (5)}$$

It may be denoted that the current is very small, about 3 to 5% of the full load rated current. Hence the primary copper loss is negligibly small hence I_c is called core loss or iron loss component. Hence power input W_o on no load always represent the iron losses, as copper loss is negligibly small. The iron losses are denoted as P_i and are constant for all load conditions.

$$\therefore \quad W_o = V_1 I_o \cos\phi_o = P_i = \text{iron loss} \quad \text{... (6)}$$

Example 1 : The no load current of a transformer is 10 A at a power factor 0f 0.25 lagging, when connected to 400 V, 50 Hz supply. Calculate,
a) Magnetising component of the no load current
b) Iron loss and c) Maximum value of flux in the core.
Assume primary winding turns as 500.

Solution : The given value are, = 10 A, cos = 0.25, = 400 V and f = 50 Hz

a) $I_m = I_o \sin\Phi_o$ = magnetising component
$\Phi_o = \cos^{-1}(0.25) = 75.522^o$
$\therefore$ $I_m = 10 \times \sin(75.522^o) = 9.6824$ A

b) P_i = iron loss = power input on no load
$= W_o = V_1 I_o \cos\Phi_o = 400 \times 10 \times 0.25$
$= 1000$ W

c) On no load, $E_1 = V_1 = 400$ V and $N_1 = 500$
Now $E_1 = 4.44 f \Phi_m N_1$
$\therefore$ $400 = 4.44 \times 50 \times \Phi_m \times 500$
$\therefore$ $\Phi_m = 3.6036$ mWb

Transformer on Load

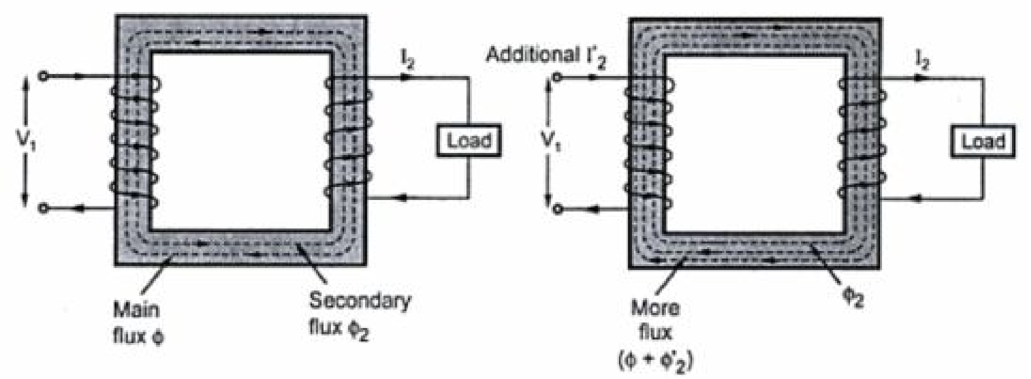
When the transformer is loaded, the current I_2 flows through the secondary winding. The magnetic and phase of I_2 is determined by the load. If load is inductive, I_2 lags V_2. If load is capacitive, I_2 leads V_2 while for resistive load, I_2 is in phase with V_2.

There exists a secondary m.m.f. $N_2 I_2$ due to which secondary current sets up its own flux Φ_2. This flux opposes the main flux Φ which is produced in the core due to magnetising component of no load current. Hence the m.m.f. is $N_2 I_2$ called demagnetising ampere-turns. This is shown in the Fig.1(a).

The flux Φ_2 momentarily reduces the main flux Φ, due to which the primary induced e.m.f. also E_1 reduces.

Hence the vector difference $\bar{V} - \bar{E}$ increases due to which primary draws more current from supply

This additional current drawn by primary is due to the load hence called load component of primary current denoted as I_2' as shown in the Fig.1(b).



(a) ϕ_2 opposes ϕ (b) Primary draws more current

Fig. 1 Transformer on load

This current I_2' is in antiphase with I_2. The current sets up its own flux Φ_2' which opposes the flux Φ_2 and helps the main fluxΦ. This flux Φ_2' neutralises the flux $\Phi_2$produced by I_2. The m.m.f. i.e. ampere turns $N_2 I_2'$ balances the ampere turns $N_2 I_2$. Hence the net flux in the core is again maintained at constant level.

Key point : Thus for any load condition, no load to full load the flux in the core is practically constant.

The load component current I_2' always neutralises the changes in the loads. Hence the transformer is called constant flux machine.

As the ampere turns are balanced we can write,

$$N_2 I_2 = N_2 I_2'$$

$$\therefore \quad I_2' = (N_2/N_1) = K I_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Thus when transformer is loaded, the primary current I_1 has two components :

1. The no load current I_o which lags V_1 by angle Φ_o. It has two components I_m and I_c.
2. The load component I_2' which in antiphase with I_2. And phase of I_2 is decided by the load.

Hence primary current I_1 is vector sum of I_o and I_2'.

$$\therefore \quad \bar{I}_1 = \bar{I}_o + \bar{I}_2 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Assume inductive load, I_2 lags E_2 by Φ_2, the phasor diagram is shown in the Fig. 2(a).

Assume purely resistive load, I_2 in phase with E_2, the phasor diagram is shown in the Fig.2(b).

Assume capacitive load, I_2 leads E_2 by Φ_2, the phasor diagram is shown in the Fig. 2(c).

Note that I_2' is always in anti phase with I_2.

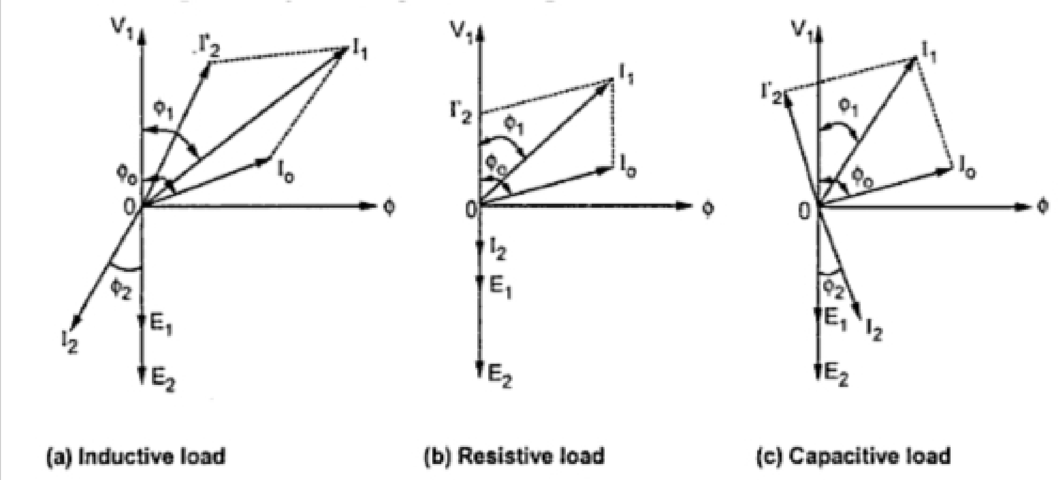


Fig. 2

Actually the phase of I_2 is with respect to V_2 i.e. angle Φ_2 is angle between I_2 and V_2. For the ideal case, E_2 is assumed equal to V_2 neglecting various drops.

The current ratio can be verified from this discussion. As the no load current I_o is very small, neglecting I_o we can write,

$$I_1 \simeq I_2'$$

Balancing the ampere turns,

$$N_1 \; I_1 = N_1 I_1 = N_2 \; I_2$$

$$\therefore \quad N_2 / N_1 = I_1 / I_2 = K$$

Under full load conditions when I_o is very small compared to full load currents, the ratio of primary and secondary current is constant.

Example : A 400/200 V transformer takes 1 A at a power factor of 0.4 on no load. If the secondary supplies a load current of 50 A at 0.8 lagging power factor, calculate the primary current.

Solution : The given values are

$$I_o = 1 \text{ A}, \cos \Phi_o = 0.4, I_2 = 50 \text{ A and } \cos \Phi_2 = .08$$

$$K = E_2/E_1 = 200/400 = 0.5$$

$$\therefore \quad I_2' = K \; I_2 = 0.5 \times 50 = 25 \text{ A}$$

The angle of I_2' is to be decided from $\cos \Phi_2 = 0.8$

Now $\cos \Phi_2 = 0.8$

$$\therefore \quad \Phi_2 = 36.86^o$$

I_2' is antiphase with I_2 which lags E_2 by 36.86^o

Consider the phasor diagram shown in the Fig. 3. The flux Φ is the reference.

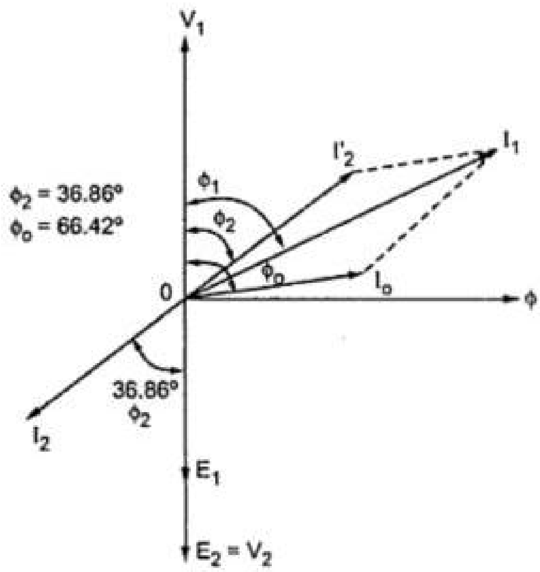


Fig. 3

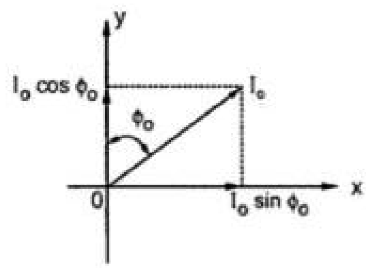


Fig. 3 (a)

Now $\cos \Phi_0 = 0.4$

$\therefore$ $\Phi_0 = 66.42^o$

$\bar{I}_1 = \bar{I}_2' + \bar{I}_0$ .... vector sum

Resolve I_0 and I_2' into two components, along reference Φ and in quadrature with Φ in phase with V_1.

x component of $I_0 = I_0 \sin \Phi_0 = 0.9165$ A

y component $I_0 = I_0 \cos \Phi_0 = 0.4$ A

$\therefore$ $\bar{I}_0 = 0.9165 + j\ 0.4$ A

x component of $I_2' = I_2' \sin \Phi_2 = 25 \sin (36.86^o) = 15$ A

y component of $I_2' = I_2' \cos \Phi_2 = 25 \times 0.8 = 20$ A

$\therefore$ $I_2' = 15 + j\ 20$ A

$\bar{I}_1 = 0.9165 + j\ 0.4 + 15 + j\ 20 = 15.9165 + j\ 20.4$ A

Thus the two components of I_1 are as shown in the Fig.3(c).

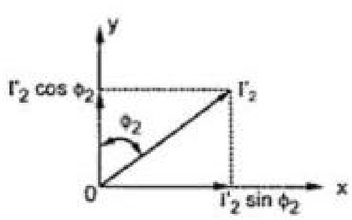


Fig. 3 (b)

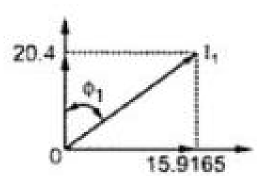


Fig. 3 (c)

$\therefore$ $I_1 = \sqrt{((15.9165)^2 + (20.4)^2)} = 25.874$ A

This is the primary current magnitude.

While $\tan \Phi_1 = 15.9165/20.4$

$\therefore$ $\Phi_1 = 37.96^o$

Hence the primary power factor is,

$\cos \Phi_1 = \cos (37.96^o) = 0.788$ lagging

Key point : Remember that Φ_1 is angle between V_1 and I_1 and as V_1 is vertical, Φ_1 is measured with respect to V_1. So do not convert rectangular to polar as it gives angle with respect to x-axis and we want it with respect to y-axis.

Equivalent Resistance

The resistance of the two windings can be transferred to any one side either primary or secondary without affecting the performance of the transformer. The transfer of the resistances on any one side is advantageous as it makes the calculations very easy. Let us see how to transfer the resistances on any one side.

The total copper loss due to both the resistances can be obtained as,

total copper loss $= I_1^2 R_1 + I_2^2 R_2$

$= I_1^2\{ R_1 + (I_2^2 / I_1^2) R_2\}$

$= I_1^2\{R_1 + (1/K^2) R_2\}$ ....(3)

Where $I_2 / I_1 = 1/K$ neglecting no load current.

Now the expression (3) indicates that the total copper loss can be expressed as $I_1^2 R_1 + I_1^2 . R_2/K^2$. This means R_2/K^2 is the resistance value of R_2 shifted to primary side which causes same copper loss with I_1 as R_2 causes with. This value of resistance which R_2/K^2 is the value of R_2 referred to primary is called equivalent resistance of secondary referred to primary. It is denoted as R_2'.

$R_2' = R_2 / K^2$ ....(4)

Hence the total resistance referred to primary is the addition of R_1 and R_2' called equivalent resistance of transformer referred to primary and denoted as R_{1e}.

$= R_1 + R_2' = R_1 + R_2 / K^2$ ....(5)

This resistance R_{1e} causes same copper loss with I_1 as the total copper loss due to the individual windings.

total copper loss $= I_1^2 R_{1e} = I_1^2 R_1 + I_2^2 R_2$ ......(6)

So equivalent resistance simplifies the calculations as we have to calculate parameters on one side only.

Similarly it is possible to refer the equivalent resistance to secondary winding.

total copper loss $= I_1^2 R_1 + I_2^2 R_2$

$= I_2^2 \{(I_1^2 / I_2^2) R_1 + R_2\}$

$= I_2^2 (K^2 R_1 + R_2)$ ....(7)

Thus the resistance $K^2 R_1$ is primary resistance referred to secondary denoted as R_1'.

$$R_1' = K^2 R_1 \quad \text{.......(8)}$$

Hence the total resistance referred to secondary is the addition of R_2 and R_1' called equivalent resistance of transformer referred to secondary and denoted as R_{2e}.

$$R_{2e} = R_2 + R_1' = R_2 + K^2 R_1 \quad \text{.........(9)}$$

total copper loss = $I_2^2 R_{2e}$ ....(10)

The concept of equivalent resistance is shown in the Fig. 1(a), (b) and (c).

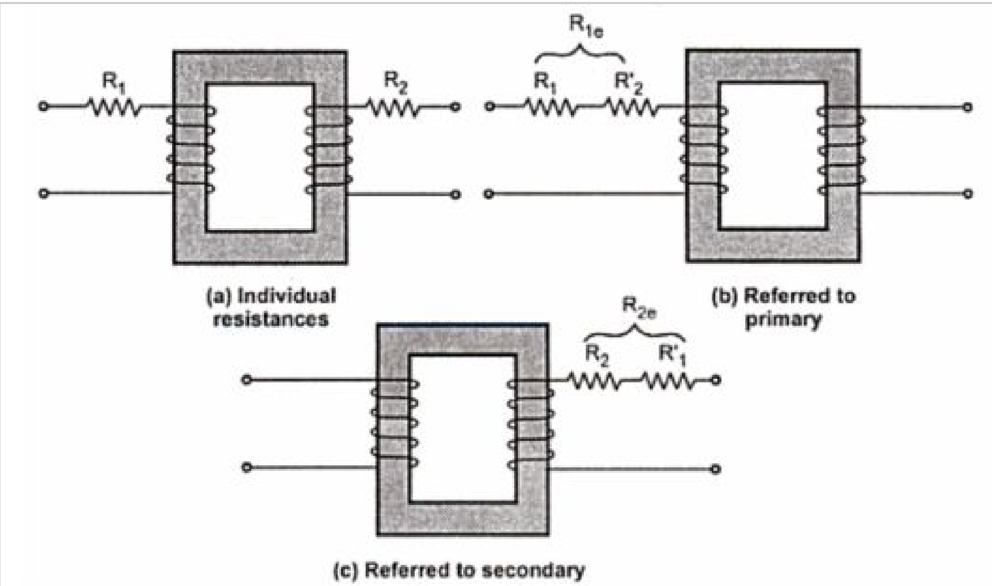


Fig. 1 Equivalent resistance

Key Point : When resistance are transferred to primary, the secondary winding becomes zero resistance winding for calculation purpose. The entire copper loss occurs due to R_{1e}. Similarly when resistances are referred to secondary, the primary becomes resistanceless for calculation purpose. The entire copper loss occurs due to R_{2e}.

Important Note : When a resistance is to be transferred from the primary to secondary, it must be multiplied by K^2. When a resistance is to be transferred from the secondary to primary, it must be divided by K^2. Remember that K is N_1 / N_2.

The result can be cross-checked by another approach. The high voltage winding is always low current winding and hence the resistance of high voltage side is high. The low voltage side is high current side and hence resistance of low voltage side is low. So while transferring resistance from low voltage side to high voltage side, its value must increase while transferring resistance from high voltage side to low voltage side, its value must decrease.

Key point :
High voltage side → Low current side → High resistance side
Low voltage side → High current side → Low resistance side

Example 1 : A 6600/400 V single phase transformer has primary resistance of 2.5 Ω and secondary resistance of 0.01 Ω calculate total equivalent resistance referred to primary and secondary.
Solution : The given values are,

$R_1 = 2.5\ \Omega \qquad R_2 = 0.01\ \Omega$

$K = 400/6600 = 0.0606$

While finding equivalent resistance referred to primary, transfer to primary as,

$R_2' = R_2/K^2 = 0.01/(0.0606)^2 = 2.7225\ \Omega$

$R_{1e} = R_1 + R_2' = 2.5 + 2.7225 = 5.2225\ \Omega$

It can be observed that primary is high voltage hence high resistance side hence while transferring from low voltage to on high voltage, its value increases.

To find total equivalent resistance referred to secondary, first calculate ,

$R_1' = K^2\ R_1 = (0.0606)^2 \times 25 = 0.00918\ \Omega$

$R_{2e} = R_2 + R_1' = 0.01 + 0.00918 = 0.01918\ \Omega$

Equivalent Leakage Reactance

Similar to the resistances, the leakage reactances also can be transferred from primary to secondary or viceversa. The relation through K^2 remains same for the transfer of recatnaces as it is studied earlier for the resistances.

Let X_1 is leakage reactance of primary and X_2 is leakage reactance of secondary.

Then the total leakage reacatance referred to primary is X_{1e} given by,

$X_{1e} = X_1 + X_2'$ where $X_2' = X_2/K^2$

While the total leakage reacatnce referred to secondary is given by ,

$X_{2e} = X_2 + X_1'$ where $X_1' = K^2 X_1$

And $K = N_2/N_1$ =transformation ratio

Equivalent Impedance

The transformer primary has resistance R_1 and reactance X_1. While the transformer secondary has resistance R_2 and reacatnce X_2. Thus we can say that the total impedance of primary winding is Z_1 which is,

$Z_1 = R_1 + j\ X_1\ \Omega$ ....(1)

And the total impedance of the secondary winding is which is ,

$Z_2 = R_2 + j\ X_2\ \Omega$ ....(2)

This is shown in the Fig. 1.

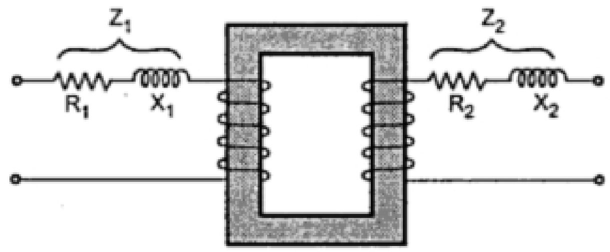


Fig. 1 Individual impedance

The individual magnitudes of and are,

$Z_1 = \sqrt{(R_1^2 + X_1^2)}$ ....(3)

and $Z_2 = \sqrt{(R_2^2 + X_2^2)}$ ....(4)

Similar to resistance and reactance, the impedance also can be referred to any one side.

Let Z_{1e} = total equivalent impedance referred to primary

then $Z_{1e} = R_{1e} + j X_{1e}$

$Z_{1e} = Z_1 + Z_2' = Z_1 + Z_2 / K^2$ ....(5)

Similarly Z_{2e} = total equivalent impedance referred to secondary

then $Z_{2e} = R_{2e} + j X_{2e}$

$Z_{2e} = Z_2 + Z_1' = Z_2 + K^2 Z_1$ ....(6)

The magnitude of Z_{1e} and Z_{2e} are,

$Z_{1e} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$ ....(7)

and $Z_{2e} = \sqrt{(R_{2e}^2 + X_{2e}^2)}$ ....(8)

It can be denoted that,

$Z_{2e} = K^2 Z_{1e}$ and $Z_{1e} = Z_{2e} / K^2$

The concept of equivalent impedance is shown in the Fig. 2.

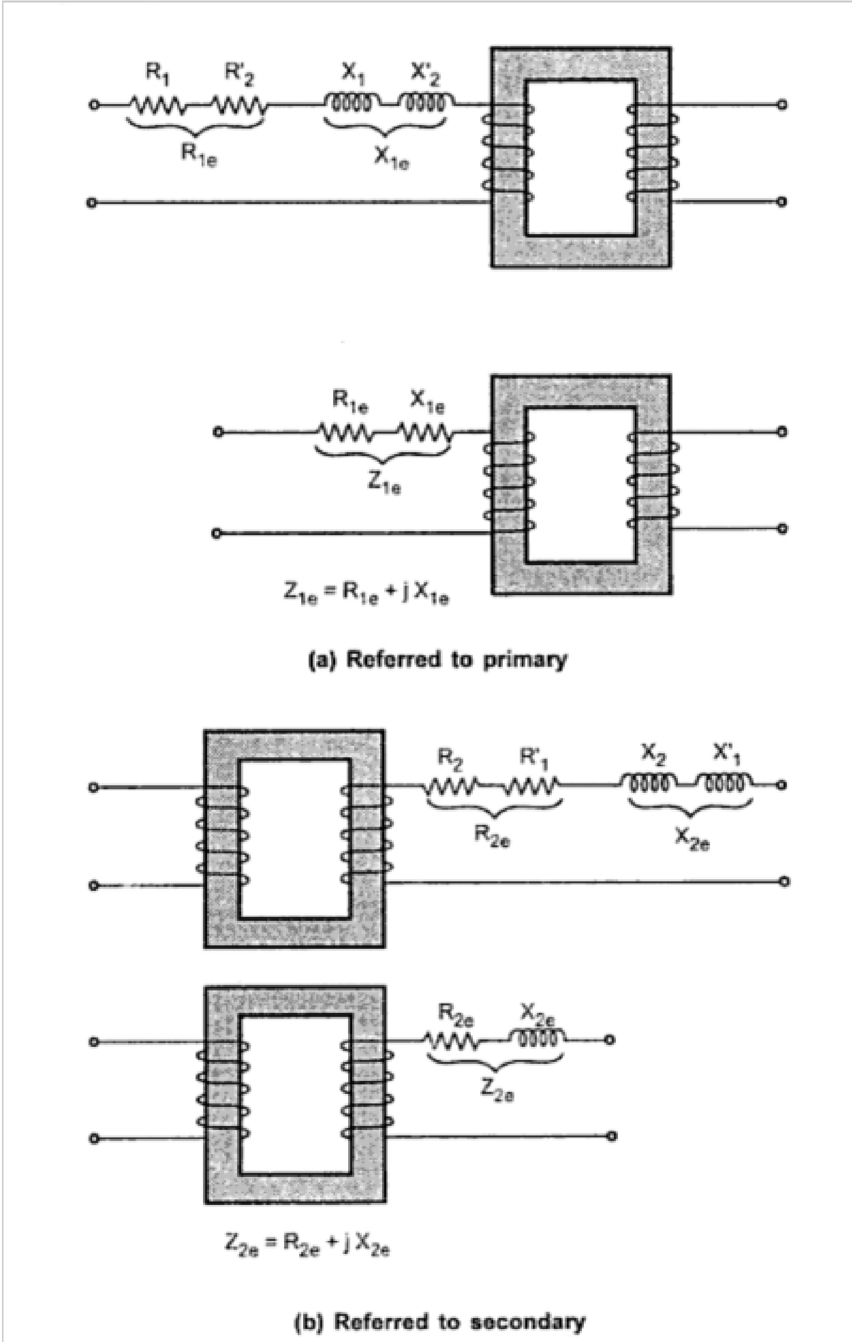


Fig 2 Equivalent impedance

Example 1 :A 15 KVA, 2200/110 V transformer has $R_1 = 1.75\Omega$, $R_2 = 0.0045\ \Omega$ the leakage reactance are $X_1 = 2.6\ \Omega$ and $X_2 = 0.0075\ \Omega$ Calculate,

a) equivalent resistance referred to primary
b) equivalent resistance referred to secondary
c) equivalent reactance referred to primary
d) equivalent reactance referred to secondary
e) equivalent impedance referred to primary

f) equivalent impedance referred to secondary
g) total copper loss

Solution : The given values are, $R_1 = 1.75\ \Omega$, $R_2 = 0.0045\Omega$, $X_1 = 2.6\ \Omega$, $X_2 = 0.0075\ \Omega$

$K = 110/2200 = 1/20 = 0.05$

a) $R_{1e} = R_1 + R_2' = R_1 + R_2/K^2 = 1.75 + 0.0045/0.05^2 = 3.55\ \Omega$

b) $R_{2e} = R_2 + R_1' = R_2 + K^2 R_1 =$
$= 0.0045 + (0.05)^2 \times 1.75 = 0.00887\ \Omega$

c) $X_{1e} = X_1 + X_2' = X_1 + X_2/K^2 = 2.6 + 0.0075/(0.05)^2 = 5.6\ \Omega$

d) $X_{2e} = X_2 + X_1' = X_2 + K^2 X_1$
$= 0.0075 + (0.05)^2 \times 2.6 = 0.014\ \Omega$

e) $Z_{1e} = R_{1e} + j X_{1e} = 3.55 + j\,5.6\ \Omega$
$Z_{1e} = \sqrt{(3.55^2 + 5.6^2)} = 6.6304\ \Omega$

f) $Z_{2e} = R_{2e} + j X_{2e} = 0.00887 + j\,0.014\ \Omega$
$Z_{2e} = \sqrt{(0.00887^2 + 0.014^2)} = 0.01657\ \Omega$

g) To find the load copper loss, calculate full load current.
(I_1) F.L. $= (KVA \times 1000)/V_1 = (25 \times 1000)/2200 = 11.3636$ A
total copper loss $= ((I_1)F.L.)^2 R_{1e} = (11.3636)^2 \times 355 = 458.4194$ W
This can be checked as,
(I_2) F.L.$= (KVA \times 1000)/V_2 = (25 \times 1000/110 = 227.272$ A
total copper loss $= I_1^2 R_1 + I_2^2 R_2$
$= (11.3636)^2 \times 1.75 + (227.373)^2 \times 0.0045$
$= 225.98 + 232.4365 = 458.419$ W

1. Voltage Regulation of Transformer

Because of the voltage drop across the primary and secondary impedances it is observed that the secondary terminal voltage drops from its no load value (E_2) to load value (V_2) as load and load current increases.

This decrease in the secondary terminal voltage expressed as a fraction of the no load secondary terminal voltage is called regulation of a transformer.

The regulation is defined as change in the magnitude of the secondary terminal voltage, when full load i.e. rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage.

Let E_2 = Secondary terminal voltage on no load
V_2 = Secondary terminal voltage on given load

then mathematically voltage regulation at given load can be expressed as,

$$\%\ \text{voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100$$

The ratio (E_2 - V_2 / V_2) is called per unit regulation.

The secondary terminal voltage does not depend only on the magnitude of the load current but also on the nature of the power factor of the load. If

V_2 is determined for full load and specified power factor condition the regulation is called full load regulation.

As load current increases, the voltage drops tend to increase V_2 and drops more and more. In case of lagging power factor $V_2 < E_2$ and we get positive voltage regulation, while for leading power factor $E_2 < V_2$ and we get negative voltage regulation.

The voltage drop should be as small as possible hence less the regulation better is the performance of a transformer.

1.1 Expression for Voltage Regulation

The voltage regulation is defined as,

$$\%R = (E_2 - V_2 \ / V_2) \times 100 = (\text{Total voltage drop} / V_2) \times 100$$

The expression for the total approximate voltage drop is already derived.

Total voltage drop = $I_2 R_{2e} \cos \Phi \pm I_2 X_{2e} \sin \Phi$

Hence the regulation can be expressed as,

$$\% R = \frac{I_2 R_{2e} \cos \phi \pm I_2 X_{2e} \sin \phi}{V_2} \times 100$$

'+' sing for lagging power factor while '-' sing for leading power factor loads.

The regulation van be further expressed interms of I_1 , V_1, R_{1e} and X_{1e}.

$$V_2 / V_1 = I_1 / I_2 = K$$

$\therefore \quad V_2 = KV_1 \quad , \quad I_2 = I_1 / K$

while $\quad R_{1e} = R_{2e} / K^2$, $X_{1e} = X_{2e} / K^2$

Substituting in the regulation expression we get,

$$\% R = \frac{I_1 R_{1e} \cos \phi \pm I_1 X_{1e} \sin \phi}{V_1} \times 100$$

1.2 Zero Voltage Regulation

We have seen that for lagging power factor and unity power factor condition $V_2 < E_2$ and we get positive regulation. But as load becomes capacitive, V_2 starts increasing as load increase. At a certain leading power factor we get $E_2 = V_2$ and the regulation becomes zero. If the load is increased further, E_2 becomes less than $V_2$and we get negative regulation.

$\therefore$ for zero voltage regulation,

$$E_2 = V_2$$

$\therefore \quad E_2 - V_2 = 0$

or $\quad V_R \cos \Phi - V_x \sin \Phi = 0$ .... -ve sing as leading power factor

where $\quad V_R = I_2 R_{2e} / V_2 = I_1 R_{1e} / V_1$ and $V_x = I_2 X_{2e} / V_2 = I_1 X_{1e} / V_1$

$\therefore \quad V_R \cos \Phi = V_x \sin \Phi$

$\therefore \quad \tan \Phi = V_R / V_x$

$\therefore \quad \cos \Phi = \cos \{\tan^{-1}(V_R / Vx)\}$

This is the leading p.f. at which voltage regulation becomes zero while supplying the load.

1.3 Constants of a Transformer

From the regulation expression we can define constants of a transformer.

$\%R = ((I_2 R_{2e} \cos\Phi \pm I_2 X_{2e} \sin\Phi) / E_2) \times 100$

$= \{(I_2 R_{2e} / E_2) \cos\Phi \pm (I_2 X_{2e} / E_2) \sin\Phi\} \times 100$

The ratio $(I_2 R_{2e} / E_2)$ or $(I_1 R_{1e} / E_1)$ is called per unit resistive drop and denoted as V_R.

The ratio $(I_2 X_{2e} / E_2)$ or $(I_1 X_{1e} / E_1)$ is called per unit reactive drop and is denoted as Vx.

The terms V_R and Vx are called constants of a transformer because for the rated output I_2, E_2, R_{1e}, X_{1e}, R_{2e} , X_{2e} are constants. The regulation can be expressed interms of V_R and Vx as,

$\%R = (V_R \cos\Phi$

iuu $\pm Vx \sin\Phi) \times 100$

On load condition, $E_2 = V_2$ and $E_1 = V_1$

where V_1 and V_2 are the given voltage ratings of a transformer. Hence V_R and Vx can be expressed as,

$V_R = I_2 R_{2e} / V_2 = I_1 R_{1e} / V_1$

and

$Vx = I_2 R_{2e} / V_2 = I_1 X_{1e} / V_1$

where V_1 and V_2 are no load primary and secondary voltages,

V_R and Vx can be expressed on percentage basis as,

Percentage resistive drop = $V_R \times 100$

Percentage reactive drop = $Vx \times 100$

Key Point : Note that and are also called per unit resistance and reactance respectively.

Example 1 : 250/125 V, 5 KVA single phase transformer has primary resistance of 0.2 Ω and reactance of 0.75Ω. The secondary resistance is 0.05 Ω and reactance of 0.2Ω

i) Determine its regulation while supplying full load on 0.8 leading p.f.

ii) The secondary terminal voltage on full load 0.8 and leading p.f.

Solution : The given values are,

$R_1 = 0.2\ \Omega$, $X_1 = 0.75\ \Omega$, $R_2 = 0.05\ \Omega$, $X_2 = 0.2\ \Omega$, $\cos\Phi = 0.8$ leading

$K = E_2 / E_1 = 125/250 = 1/2 = 0.5$

(I_2) F.L. $= KVA/V_2 = 5\times10^3 / 125 = 40$ A ...... full load

$R_{2e} = R_2 + K^2 R_1 = 0.05 + (0.5)^2 \times 0.2 = 0.1\ \Omega$

$X_{2e} = X_2 + K^2 X_1 = 0.2 + (0.5)^2 \times 0.75 = 0.3875\ \Omega$

i) Regulation on full load, $\cos\Phi = 0.8$ leading

$\sin\Phi = 0.6$

$\therefore$ $\%R = ((I_2 R_{2e} \cos\Phi - I_2 X_{2e} \sin\Phi) / E_2) \times 100$

where I_2 = Full load current

$\therefore$ $\%R = ((40 \times 0.1 \times 0.8 - 40 \times 0.3875 \times 0.6)/125) \times 100 = -4.88\%$

ii) For secondary terminal voltage, use basic expression of regulation

$\%R = ((E_2 - V_2)/E_2) \times 100$

$\therefore$ $-4.88 = ((125 - V_2)/125) \times 100$

$\therefore$ $-6.1 = 125 - V_2$

$\therefore$ $V_2 = 131.1$ V

It can be seen that for leading p.f. $E_2 < V_2$.

Example 2 : Calculate the regulation of a transformer in which the copper loss is 1% of output and the percentage reactance drop is 5% when load power factor is
i) 0.9 lagging and ii) 0.9 leading.
Solution : Given values are,

$\%X = 5\%$

Now copper loss is, $P_{cu} = I_2^2 R_{2e}$

and output is, $P_{out} = V_2 I_2$

$\therefore$ $\% \text{ Copper loss} = (P_{cu}/P_{out}) \times 100 = (I_2^2 R_{2e} / V_2 I_2) \times 100$

$\% V_R = (I_2 R_{2e} / V_2) \times 100$

$= (I_2 R_{2e} / V_2) \times (I_2 / I_2) \times 100 = (I_2^2 R_{2e} / V_2 I_2) \times 100$

$= \%$ copper loss

$\therefore$ $V_R = 1\% = 0.01$ and $V_x = 5\% = 0.05$

i) $\cos \Phi = 0.9$ lagging

$\therefore$ $\sin \Phi = 0.4358$

$\therefore$ $\% R = (V_R \cos \Phi + V_x \sin \Phi) \times 100 = (0.01 \times 0.9 + 0.05 \times 0.4358) \times 100$

$= + 3.08\%$

ii) $\cos \Phi = 0.9$ leading

$\therefore$ $\% R = (V_R \cos \Phi - V_x \sin \Phi) \times 100 = (0.01 \times 0.9 - 0.05 \times 0.4358) \times 100$

$= -1.28\%$

Losses in a Transformer

In a transformer, there exists two types of losses.
i) The core gets subjected to an alternating flux, causing core losses.
ii) The windings carry currents when transformer is loaded, causing copper losses.

Core or Iron Losses

Due to alternating flux set up in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is loss of energy in this process which is called hysteresis loss.

It is given by, hysteresis loss $= K_h B_m^{1.67} f v$ watts

where K_h = Hysteresis constant depends on material.
B_m = Maximum flux density.
f = Frequency.
v = Volume of the core.

The induced e.m.f. in the core tries to set up eddy currents in the core and hence responsible for the eddy current losses. The eddy current loss is given by, Eddy current loss = $K_e B_m^2 f^2 t^2$ watts/ unit volume

where K_e = Eddy current constant

t = Thickness of the core

As seen earlier, the flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant. Hence the flux density B_m in the core and hence both hysteresis and eddy current losses are constants at all the loads. Hence the core or iron losses are also called constant losses. The iron losses are denoted as P_i.

The iron losses are minimized by using high grade core material like silicon steel having very low hysteresis loop by manufacturing the core in the form of laminations.

Copper Losses

The copper losses are due to the power wasted in the form of $I^2 R$ loss due to the resistances of the primary and secondary windings. The copper loss depends on the magnitude of the currents flowing through the windings.

Total Cu loss = $I_1^2 R_1 + I_2^2 R_2 = I_1^2 (R_1 + R_2')= I_2^2 (R_2 + R_1')$

$= I_1^2 R_{1e} = I_2^2 R_{2e}$

The copper looses are denoted as. If the current through the windings is full load current, we get copper losses at full load. If the load on transformer is half then we get copper losses at half load which are less than full load copper losses. Thus copper losses are called variable losses. For transformer VA rating is or. As is constant, we can say that copper losses are proportional to the square of the KVA rating.

So, $P_{cu} \alpha I^2 \alpha (KVA)^2$

Thus for a transformer,

Total losses = Iron losses + Copper losses

$= P_i + P_{cu}$

Key point : It is seen that the iron losses depend on the supply voltage while the copper losses depend on the current. The losses are not dependent on the phase angle between voltage and current. Hence the rating of the transformer is expressed as a product of voltage and current and called VA rating of transformer. It is not expressed in watts or kilo watts. Most of the times, rating is expressed in KVA.

Efficiency of a Transformer

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

∴ Power output = Power input - Total losses

∴ Power input = Power output + Total losses

= Power output + $P_i + P_{cu}$

The efficiency of any device is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expresses as,

η = Power output/power input

∴ η = Power output/(power output + $P_i + P_{cu}$)

Now power output = $V_2 I_2 \cos \Phi$

where cos Φ = Load power factor

The transformer supplies full load of current I_2 and with terminal voltage V_2.

P_{cu} = Copper losses on full load = $I_2^2 R_{2e}$

$\therefore$ $\eta = (V_2 I_2 \cos \Phi_2)/(V_2 I_2 \cos \Phi_2 + P_i + I_2^2 R_{2e})$

But $V_2 I_2$ = VA rating of a transformer

$\therefore$ η = (VA rating x cos Φ) / (VA rating x cos Φ + P_i + $I_2^2 R_{2e}$)

$$\therefore \quad \% \eta = \frac{(\text{VA rating}) \times \cos\phi}{(\text{VA rating}) \times \cos\phi + P_i + I_2^2 R_{2e}} \times 100$$

This is full load percentage efficiency with,

I_2 = Full load secondary current

But if the transformer is subjected to fractional load then using the appropriate values of various quantities, the efficiency can be obtained.

Let n =Fraction by which load is less than full load = Actual load/Full load

For example, if transformer is subjected to half load then,

n = Half load/Full load = (1/2)/2 = 0.5

when load changes, the load current changes by same proportion.

$\therefore$ new I_2 = n (I_2) F.L.

Similarly the output $V_2 I_2 \cos\Phi_2$ also reduces by the same fraction. Thus fraction of VA rating is available at the output.

Similarly as copper losses are proportional to square of current then,

new P_{cu} = n^2 (P_{cu}) F.L.

Key Point : So copper losses get reduced by n^2.

In general for fractional load the efficiency is given by,

$$\% \eta = \frac{n\,(\text{VA rating}) \cos\phi}{n\,(\text{VA rating}) \cos\phi + P_i + n^2\,(P_{cu})\,\text{F.L.}} \times 100$$

where n = Fraction by which load power factor lagging, leading and unity the efficiency expression does not change, and remains same.

Example : A 4 KVA, 200/400 V, 50 Hz, single phase transformer has equivalent resistance referred to primary as 0.15 Ω Calculate,

i) The total copper losses on full load.

ii) The efficiency while supplying full load at 0.9 p.f. lagging.

iii) The efficiency while supplying half load at 0.8 p.f. leading.

Assume total iron losses equal to 60 W.

Solution : The given values are,

V_1 = 200 V, V_2 = 400 V, S = 4 KVA, R_{1e}= 0.15 Ω , P_i = 60 W

K = 400/200 = 2

$\therefore$ $R_{2e} = K^2 R_{1e} = (2)^2 \times 0.15 = 0.6\ \Omega$

(I_2)F.L. = KVA/V_2 = 4 x 10^3/400 = 10 A

(i) Total copper losses on full load,

(P_{cu})F.L. = $\{(I_2)\text{ F.L.}\}^2 R_{2e} = (10)^2 \times 0.6 = 60$ W

(ii) cos Φ = 0.9 lagging and full load

$$\therefore \quad \% \eta = \frac{\text{VA rating} \cos\phi}{\text{VA rating} \cos\phi + P_i + (P_{cu}) \text{F.L.}} \times 100$$

$$\therefore \quad \eta = \frac{4\times10^3 \times 0.9}{4\times10^3\times0.9+60+60} \times 100 = \mathbf{96.77\ \%}$$

(iii) $\cos \Phi = 0.8$ leading, half load

As half load, $n = 0.5$

$$\therefore \quad \% \eta = \frac{n \times (\text{VA rating}) \cos\phi}{n \times (\text{VA rating}) \cos\phi + P_i + (P_{cu}) \text{H.L.}} \times 100$$

$$= \frac{0.5\times4\times10^3\times0.8}{0.5\times4\times10^3\times0.8+60+15} \times 100 = \mathbf{95.52\ \%}$$

Condition for Maximum Efficiency

When a transformer works on a constant input voltage and frequency then efficiency varies with the load. As load increases, the efficiency increases. At a certain load current, it achieves a maximum value. If the transformer is loaded further the efficiency starts decreasing. The graph of efficiency against load current I_2 is shown in the Fig.1

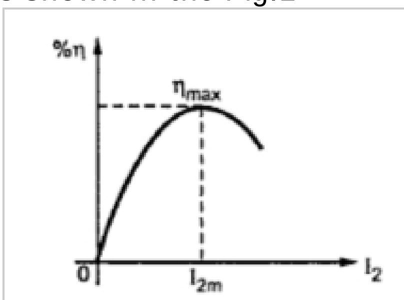


Fig. 1

The load current at which the efficiency attains maximum value is denoted as I_{2m} and maximum efficiency is denoted as η_{max}.

Let us determine,

1. Condition for maximum efficiency.
2. Load current at which η_{max} occurs.
3. KVA supplied at maximum efficiency.

The efficiency is a function of load i.e. load current I_2 assuming $\cos \Phi$ constant. The secondary terminal voltage V_2 is also assumed constant. So for maximum efficiency,

$d\eta / d I_2 = 0$

Now $\eta = (V_2 I_2 \cos \Phi_2) / (V_2 I_2 \cos \Phi_2 + P_i + I_2^2 R_{2e})$

$$\therefore \quad \frac{d\eta}{dI_2} = \frac{d}{dI_2}\left[\frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}}\right] = 0$$

$$\therefore (V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) \frac{d}{dI_2}(V_2 I_2 \cos\phi_2)$$

$$-(V_2 I_2 \cos\phi_2)\cdot\frac{d}{dI_2}(V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{2e}) = 0$$

$\therefore \quad (V_2 I_2 \cos \Phi_2 + P_i + I_2^2 R_{2e})(V_2 \cos \Phi_2) - (V_2 I_2 \cos \Phi_2)(V_2 \cos \Phi_2 + 2I_2 R_{2e}) = 0$

Cancelling $(V_2 \cos \Phi 2)$ from both the terms we get,

$V_2 I_2 \cos \Phi_2 + P_i + I_2^2 R_{2e} - V_2 I_2 \Phi_2 - 2I_2^2 R_{2e} = 0$

$\therefore \quad P_i - I_2^2 R_{2e} = 0$

$\therefore \quad P_i = I_2^2 R_{2e} = P_{cu}$

So condition to achieve maximum efficiency is that,

Copper losses = Iron losses

1.1 Load Current I_{2m} at Maximum Efficiency

For η_{max}, $\quad I_2^2 R_{2e} = P_i$ but $I_2 = I_{2m}$

$I_{2m}^2 R_{2e} = P_i$

$I_{2m} = \sqrt{(P_i / R_{2e})}$

This is the load current at η_{max},

Let $\quad (I_2)F.L. =$ Full load current

$\therefore \quad I_{2m} / (I_2) F.L. = (1/(I_2) F.L.)\sqrt{(P_i / R_{2e})}$

$\therefore \quad I_{2m} / (I_2) F.L. = \sqrt{(P_i)/(\{(I_2) F.L.\}^2 R_{2e})}$

$= \sqrt{(P_i)/((P_{cu}) F.L.)}$

$\therefore \quad I_{2m} = (I_2)F.L.\sqrt{(P_i)/((P_{cu}) F.L.)}$

This is the load current at η_{max} interms of full load current.

1.2 KVA supplied at maximum Efficiency

For constant V_2 the KVA supplied is the function of.

KVA at $\eta_{max} = I_{2m} V_2 = V_2 (I_2)$ F.L. x $\sqrt{(P_i)/((P_{cu})F.L.)}$

KVA at η_{max} = (KVA rating) x $\sqrt{(P_i)/((P_{cu})F.L.)}$

Substituting condition for in the expression of efficiency, we can write expression for η_{max} as,

$$\%\,\eta_{max} = \frac{V_2 I_{2m} \cos\phi}{V_2 I_{2m} \cos\phi + 2P_i} \times 100 \qquad \text{as } P_{cu} = P_i$$

$$\text{i.e.} \qquad \%\,\eta_{max} = \frac{\text{kVA for}\,\eta_{max} \cos\phi}{\text{kVA for}\,\eta_{max} \cos\phi + 2P_i}$$

Example : A 250 KVA single phase transformer has iron loss of 1.8 KW. The full load copper loss is 2000 watts. Calculate

i) Efficiency at full load, 0.8 lagging p.f.

ii) KVA supplied at maximum eficiency

iii) Maximum efficiency at 0.8 lagging p.f.

Solution : The given values are,

P_i = 1800 W , (P_{cu})F.L. = 2000 W

i)

$$\% \eta = \frac{(\text{VA rating}) \cos\phi}{(\text{VA rating}) \cos\phi + P_i + (P_{cu}) \text{F.L.}} \times 100$$

$$= \frac{250 \times 10^3 \times 0.8}{250 \times 10^3 \times 0.8 + 1800 + 2000} \times 100$$

= 98.135%

ii) KVA at = KVA rating x $\sqrt{(P_i) / ((P_{cu})\text{F.L.})}$

= 250 x $\sqrt{(1800/2000)}$

= 237.1708 KVA

iii)

$$\% \eta_{max} = \frac{\text{kVA at}\,\eta_{max} \times \cos\phi}{\text{kVA at}\,\eta_{max} \times \cos\phi + P_i + P_i} \times 100$$

where $P_{cu} = P_i$ =1800W

$$\therefore \quad \% \eta_{max} = \frac{237.1708 \times 10^3 \times 0.8}{237.1708 \times 10^3 \times 0.8 + 2 \times 1800} \times 100$$

= 98.137%

O.C. and S.C. Tests on Single Phase Transformer

The efficiency and regulation of a transformer on any load condition and at any power factor condition can be predetermined by indirect loading method. In this method, the actual load is not used on transformer. But the equivalent circuit parameters of a transformer are determined by conducting two tests on a transformer which are,

1. Open circuit test (O.C Test)
2. Short circuit test (S.C.Test)

The parameters calculated from these test results are effective in determining the regulation and efficiency of a transformer at any load and power factor condition, without actually loading the transformer. The advantage of this method is that without much power loss the tests can be performed and results can be obtained. Let us discuss in detail how to perform these tests and how to use the results to calculate equivalent circuit parameters.

1.1 Open Circuit Test (O.C. Test)

The experimental circuit to conduct O.C test is shown in the Fig. 1.

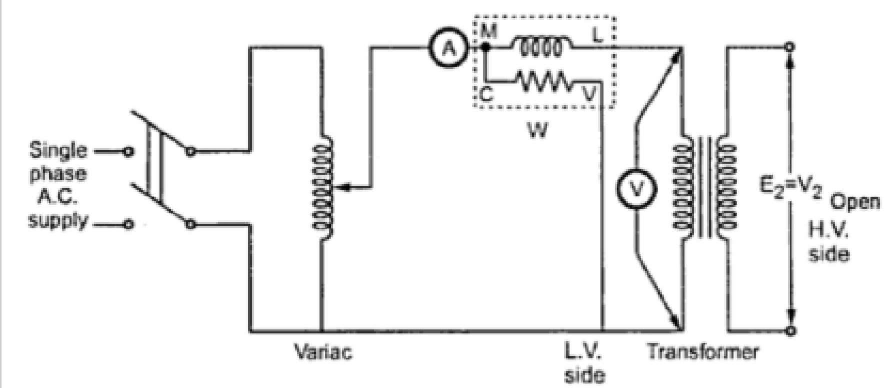


Fig 1. Experimental circuit for O.C. test

The transformer primary is connected to a.c. supply through ammeter, wattmeter and variac. The secondary of transformer is kept open. Usually low voltage side is used as primary and high voltage side as secondary to conduct O.C test.

The primary is excited by rated voltage, which is adjusted precisely with the help of a variac. The wattmeter measures input power. The ammeter measures input current. The voltemeter gives the value of rated primary voltage applied at rated frequency.

Sometimes a voltmeter may be connected across secondary to measure secondary voltage which is $V_2 = E_2$ when primary is supplied with rated voltage. As voltmeter resistance is very high, though voltmeter is connected, secondary is treated to be open circuit as voltmeter current is always negligibly small.

When the primary voltage is adjusted to its rated value with the help of variac, readings of ammeter and wattmeter are to be recorded.
The observation table is as follows

V_o volts	I_o amperes	W_o watts
Rated		

V_o = Rated voltage
W_o = Input power
I_o = Input current = no load current

As transformer secondary is open, it is on no load. So current drawn by the primary is no load current I_o. The two components of this no load current are,

$I_m = I_o \sin \Phi_o$
$I_c = I_o \cos \Phi_o$

where $\cos \Phi_o$ = No load power factor
And hence power input can be written as,

$W_o = V_o I_o \cos \Phi_o$

The phasor diagram is shown in the Fig. 2.

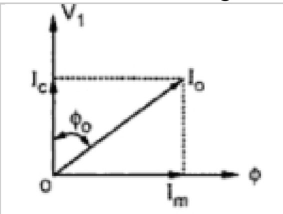


Fig. 2

As secondary is open, $I_2 = 0$. Thus its reflected current on primary is also zero. So we have primary current $I_1 = I_o$. The transformer no load current is always very small, hardly 2 to 4 % of its full load value. As $I_2 = 0$, secondary copper losses are zero. And $I_1 = I_o$ is very low hence copper losses on primary are also very very low. Thus the total copper losses in O.C. test are negligibly small. As against this the input voltage is rated at rated frequency hence flux density in the core is at its maximum value. Hence iron losses are at rated voltage. As output power is zero and copper losses are very low, the total input power is used to supply iron losses. This power is measured by the wattmeter i.e. W_o. Hence the wattmeter in O.C. test gives iron losses which remain constant for all the loads.

$\therefore \quad W_o = P_i$ = Iron losses

Calculations : We know that,

$W_o = V_o I_o \cos \Phi$

$\cos \Phi_o = W_o / (V_o I_o)$ = no load power factor

Once $\cos \Phi_o$ is known we can obtain,

$I_c = I_o \cos \Phi_o$

and $I_m = I_o \sin \Phi_o$

Once I_c and I_m are known we can determine exciting circuit parameters as,

$R_o = V_o / I_c \quad \Omega$

and $X_o = V_o / I_m \quad \Omega$

Key Point : The no load power factor $\cos \Phi_o$ is very low hence wattmeter used must be low power factor type otherwise there might be error in the results. If the meters are connected on secondary and primary is kept open then from O.C. test we get R_o' and X_o' with which we can obtain R_o and X_o knowing the transformation ratio K.

1.2 Short Circuit Test (S.C. Test)

In this test, primary is connected to a.c. supply through variac, ammeter and voltmeter as shown in the Fig. 3.

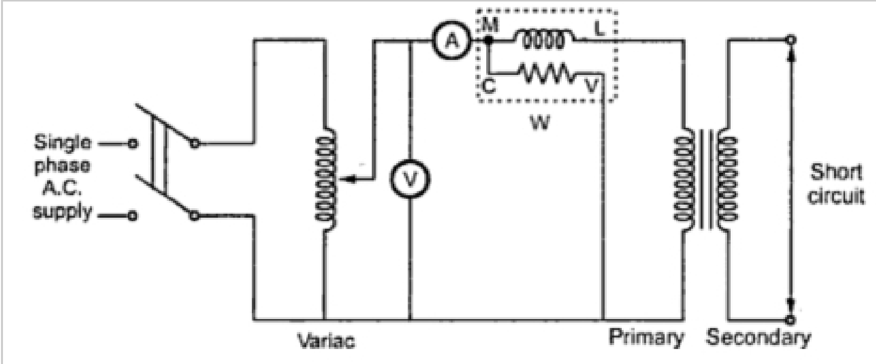


Fig. 3 Fig 1. Experimental circuit for O.C. test

The secondary is short circuited with the help of thick copper wire or solid link. As high voltage side is always low current side, it is convenient to connect high voltage side to supply and shorting the low voltage side.

As secondary is shorted, its resistance is very very small and on rated voltage it may draw very large current. Such large current can cause overheating and burning of the transformer. To limit this short circuit current, primary is supplied with low voltage which is just enough to cause rated current to flow through primary which can be observed on an ammeter. The low voltage can be adjusted with the help of variac. Hence this test is also called low voltage test or reduced voltage test. The wattmeter reading as well as voltmeter, ammeter readings are recorded. The observation table is as follows,

V_{sc} volts	I_{sc} amperes	W_{sc} watts
	Rated	

Now the current flowing through the windings are rated current hence the total copper loss is full load copper loss. Now the voltage supplied is low which is a small fraction of the rated voltage. The iron losses are function of applied voltage. So the iron losses in reduced voltage test are very small. Hence the wattmeter reading is the power loss which is equal to full load copper losses as iron losses are very low.

∴ $W_{sc} = (P_{cu})$ F.L. = Full load copper loss

Calculations : From S.C. test readings we can write,

$W_{sc} = V_{sc} I_{sc} \cos \Phi_{sc}$

∴ $\cos \Phi sc = V_{sc} I_{sc} / W_{sc}$ = short circuit power factor

$W_{sc} = I_{sc}^2 R_{1e}$ = copper loss

∴ $R_{1e} = W_{sc} / I_{sc}^2$

while $Z_{1e} = V_{sc} / I_{sc} = \sqrt{(R_{1e}^2 + X_{1e}^2)}$

∴ $X_{1e} = \sqrt{(Z_{1e}^2 - R_{1e}^2)}$

Thus we get the equivalent circuit parameters R_{1e}, X_{1e} and Z_{1e}. Knowing the transformation ratio K, the equivalent circuit parameters referred to secondary also can be obtained.

Important Note : If the transformer is step up transformer, its primary is L.V. while secondary is H.V. winding. In S.C. test, supply is given to H.V. winding and L.V is shorted. In such case we connect meters on H.V. side which is transformer secondary through for S.C. test purpose H.V side acts as primary. In such case the parameters calculated from S.C. test readings are referred to secondary which are R_{2e}, Z_{2e} and X_{2e}. So before doing calculations it is necessary to find out where the readings are recorded on transformer primary or secondary and accordingly the parameters are to be determined. In step down transformer, primary is high voltage itself to which supply is given in S.C. test. So in such case test results give us parameters referred to primary i.e. R_{1e}, Z_{1e} and X_{1e}.

Key point : In short, if meters are connected to primary of transformer in S.C. test, calculations give us R_{1e} and Z_{1e} if meters are connected to secondary of transformer in S.C. test calculations give us R_{2e} and Z_{2e}.

1.3 Calculation of Efficiency from O.C. and S.C. Tests

We know that,

From O.C. test, $W_o = P_i$

From S.C. test, $W_{sc} = (P_{cu})$ F.L.

$$\therefore \quad \% \eta \text{ on full load} = \frac{V_2 (I_2) \text{ F.L.} \cos\phi}{V_2 (I_2) \text{ F.L. } \cos + W_o + W_{sc}} \times 100$$

Thus for any p.f. cos Φ_2 the efficiency can be predetermined. Similarly at any load which is fraction of full load then also efficiency can be predetermined as,

$$\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos\phi}{n \times (\text{VA rating}) \times \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where n = fraction of full load

or
$$\% \eta = \frac{n V_2 I_2 \cos\phi}{n V_2 I_2 \cos\phi + W_o + n^2 W_{sc}} \times 100$$

where $I_2 = n (I_2)$ F.L.

1.4 Calculation of Regulation

From S.C. test we get the equivalent circuit parameters referred to primary or secondary.

The rated voltages V_1, V_2 and rated currents (I_1) F.L. and (I_2) F.L. are known for the given transformer. Hence the regulation can be determined as,

$$\% R = \frac{I_2 R_{2e} \cos\phi \pm I_2 X_{2e} \sin\phi}{V_2} \times 100$$

$$= \frac{I_1 R_{1e} \cos\phi \pm I_1 X_{1e} \sin\phi}{V_1} \times 100$$

where I_1, I_2 are rated currents for full load regulation.
For any other load the currents I_1, I_2 must be changed by fraction n.
∴ I_1, I_2 at any other load = n (I_1) F.L., n (I_2) F.L.

Key Point : Thus regulation at any load and any power factor can be predetermined, without actually loading the transformer.

Example 1 : A 5 KVA, 500/250 V, 50 Hz, single phase transformer gave the following readings,
O.C. Test : 500 V, 1 A, 50 W (L.V. side open)
S.C. Test : 25 V, 10 A, 60 W (L.V. side shorted)
Determine : i) The efficiency on full load, 0.8 lagging p.f.
ii) The voltage regulation on full load, 0.8 leading p.f.
iii) The efficiency on 60% of full load, 0.8 leading p.f.
iv) Draw the equivalent circuit referred to primary and insert all the values in it.

Solution : In both the tests, meters are on H.V. side which is primary of the transformer. Hence the parameters obtained from test results will be referred to primary.

From O.C. test, V_o = 500 V, I_o = 1 A, W_o= 50 W
∴ $\cos \Phi_o = W_o/V_o I_o = 50/(500x1) = 0.1$
∴ $I_c = I_o \cos = 1 \times 0.1 = 0.1$ A
and $I_m = I_o \sin \Phi_o = 1 \times 0.9949 = 0.9949$ A
∴ $R_o = V_o / I_c = 500/0.1 = 5000\ \Omega$
and $X_o = V_o/I_m = 500/0.9949 = 502.52\ \Omega$
and $W_o = P_i$= iron losses = 50 W
From S.C. test, V_{sc} = 25 V, I_{sc} = 10 A, W_{sc} = 60 W
∴ $R_{1e} = W_{sc} / I_{sc}^2 = 60/(10)^2 = 0.6\ \Omega$
$Z_{1e} = V_{sc} / I_{sc} = 25/10 = 2.5\ \Omega$
∴ $X_{1e}= \sqrt{(2.5^2 - 0.6^2)} = 2.4269\ \Omega$
(I_1) F.L. = VA rating/V_1
$= (5 \times 10^3)/500 = 10$ A
and $I_{sc} = (I_1)$ F.L.
∴ $W_{sc} = (P_{cu})$ F.L. = 60 W

i) η on full load, cos = 0.8 lagging

$$\% \eta = \frac{(\text{VA rating}) \cos\phi_2}{(\text{VA rating}) \cos\phi_2 + P_i + (P_{cu}) \text{F. L.}} \times 100$$

$$= \frac{5\times10^3 \times 0.8}{5\times10^3 \times 0.8 + 50 + 60} \times 100 = \mathbf{97.32\ \%}$$

ii) Regulation on full load, $\cos \Phi_2$ = 0.8 leading

$$\% R = \frac{(I_1) F. L. R_{1e} \cos\phi - (I_1) F. L. X_{1e} \sin\phi}{V_1} \times 100$$

$$= \frac{10 \times 0.6 \times 0.8 - 10 \times 2.4269 \times 0.6}{500} \times 100$$

$= -1.95\ \%$

iii) For 60% of full load, n = 0.6 and $\cos \Phi_2 = 0.8$ leading]

$\therefore$ P_{cu} = copper loss on new load = $n^2 \times (P_{cu})$ F.L.

$= (0.6)^2 \times 60 = 21.6$ W

= 97.103 %

iv) The equivalent circuit referred to primary is shown in the Fig. 4.

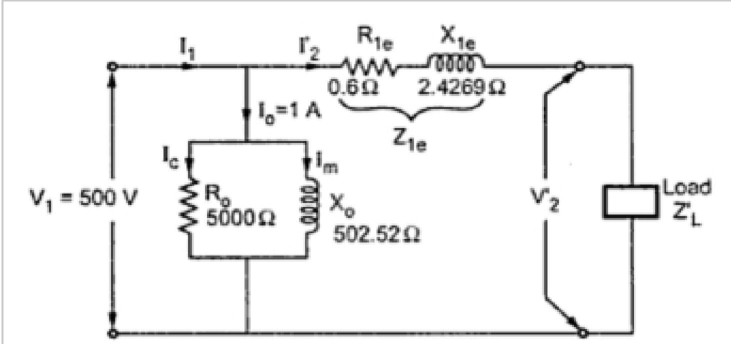


Fig. 4

Example 2 : The open circuit and short circuit tests on a 10 KVA, 125/250 V, 50 Hz, single phase transformer gave the following results :

O.C. test : 125 V, 0.6 A, 50 W (on L.V. side)

S.C. test : 15 V, 30 A. 100 W (on H.V. side)

Calculate : i) copper loss on full load

ii) full load efficiency at 0.8 leading p.f.

iii) half load efficiency at 0.8 leading p.f.

iv) regulation at full load, 0.9 leading p.f.

Solution : From O.C. test we can weite,

$W_o = P_i = 50$ W = Iron loss

From S.C. test we can find the parameters of equivalent circuit. Now S.C. test is conducted on H.V. side i.e. meters are on H.V. side which is transformer secondary. Hence parameters from S.C. test results will be referred to secondary.

$V_{sc} = 15$ V, $I_{sc} = 30$ A, $W_{sc} = 100$ W

$\therefore$ $R_{2e} = W_{sc}/(I_{sc})^2 = 10/(30)^2 = 0.111\Omega$

$Z_{1e} = V_{sc}\ /I_{sc} = 15/30 = 0.5\ \Omega$

$\therefore$ $X_{2e} = \sqrt{(Z_{2e}^2 - R_{2e}^2)} = 0.4875\ \Omega$

i) Copper loss on full load

(I_2) F.L. = VA rating/V_2 = $(10 \times 10^3)/250 = 40$ A

In short circuit test, $I_{sc} = 30$ A and not equal to full load value 40 A.

Hence W_{sc} does not give copper loss on full load

$\therefore$ $W_{sc} = P_{cu}$ at 30 A = 100 W

Now $P_{cu} \propto I^2$

(P_{cu} at 30 A)/(P_{cu} at 40 A) = $(30/40)^2$
100/(P_{cu} at 40 A) = 900/1600
P_{cu} at 40 A = 177.78 W
∴ (P_{cu}) F.L. = 177.78 W
ii) Full load η , cos Φ_2 = 0.8

$$\% \eta \text{ on full load } = \frac{V_2(I_2)\text{ F. L. } \cos\phi_2}{V_2(I_2)\text{ F. L.}\cos\phi_2 + P_i + (P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{250\times40\times0.8}{250\times40\times0.8+50+177.78}\times100 = \mathbf{97.23\ \%}$$

iii) Half load η , cos Φ_2 = 0.8
n = 0.5 as half load, (I_2) H.L. = 0.5 x 40 = 20

$$\therefore \ \% \eta \text{ on half load } = \frac{V_2(I_2)\text{ H. L. } \cos\phi_2}{V_2(I_2)\text{ H. L.}\cos\phi_2 + P_i + n^2(P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{n\text{ (VA rating)} \cos\phi_2}{n\text{ (VA rating)}\cos\phi_2 + P_i + n^2(P_{cu})\text{ F. L.}} \times 100$$

$$= \frac{0.5\times10\times10^3\times0.8}{0.5\times10\times10^3\times0.8+50+(0.5)^2\times177.78}\times100$$

= 97.69%

iv) Regulation at full load, cos Φ = 0.9 leading

$$\% R = \frac{(I_2)\text{ F.L.}R_{2e}\cos\phi-(I_2)\text{ F.L. }X_{2e}\sin\phi}{V_2}\times100$$

$$= \frac{40\times0.111\times0.9-40\times0.4875\times0.4358}{250}\times100$$

= -1.8015%

SECTION – A
Objective Questions

1. Which of the following does not change in a transformer?
 a) Voltage b)Current c)frequency d)all of the above

2. A transformer is laminated to
 a) reduce hysteresis loss
 b) reduce eddy current loss
 c) reduce copper loss\
 d)reduce all the above losses

3. The path of magnetic flux in a transformer should have
 a) high resistance

b) high reluctance
c) low resistance
d) low reluctance

4. Primary winding of a transformer
a) is always a low voltage winding
b) is always a high voltage winding
c) could either be a high voltage winding or a low voltage winding
d) none of the above

5. Which winding in a transformer has more number of turns?
a) Primary winding
b) secondary winding
c) low voltage winding
d) High voltage winding

6. A transformer
a) changes AC to DC
b) changes DC to AC
c) Steps up or down DC voltages and currents
d) steps up or down AC voltages and currents

7. The rating of a transformer is expressed in
a) KW
b)KVA
c)KVAR
d) none of the above

8. What happens when the primary of a transformer is given a DC supply?
a) The transformer may start to smoke and burn
b) No effect
c) transformer may operate at low efficiency
d) transformer may operate at a high effifciency

9. A step up transformer
a) Step up the level of voltage
b) Step down the level of current
c) Step up the level of power
d) step up the level of frequency
e) only a and b

10. Open circuit is conducted to determine
a) Hysteris losses
b) Core losses

c) Eddy current losses
d) Copper losses

11. During open circuit test on a transformer
    a) Primary is supplied with rated voltage
    b) Primary is supplied full load current
    c) Primary is supplied current at reduced voltage
    d) Primary is supplied with rated KVA

12. Maximum efficiency occurs when copper loss is ______ iron loss
    a) equal to the
    b) greater than the
    c) Less than the
    d) none of the above

13. The primary and secondary windings of a transformer are
    a) Inductively coupled
    b) conductively coupled
    c) electrically coupled
    d) mechanically coupled

SECTION – B
Descriptive Questions

1. a) What is the efficiency of a transformer? Obtain the condition for maximum efficiency.
   b) List out various losses in transformer.
2. Explain the procedure for conducting open circuit and Short circuit test on transformers.
3. Define voltage regulation. Derive an expression for voltage regulation of a single phase transformer.

4. Explain why the rating of transformer is always in KVA.
5. What is a transformer? Explain the operation of a single-phase transformer.
6. Derive the EMF equation of a single phase transformer?

7. A 25 KVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz supply. Find the full load primary and secondary currents, the secondary e.m.f and maximum flux in the core. Neglect leakage drops and no-load primary current.

8. The maximum efficiency of a 100 KVA, 6600/250 V, 50 Hz, 1-phase transformer occurs at half load and is 98 % at unity power factor. Calculate the full load efficiency at 0.8 power factor lagging.

9. The maximum flux density in the core of 250/3000V, 50Hz single phase transformer is 1.2wb/m2. If the emf per turn is 8V, determine i) Primary and secondary turns ii) Area of the core.
10. A 2000/200 V, 20 kVA transformer has 66 turns in the secondary. Calculate i) primary turns ii) primary and secondary full-load currents.
11. A single phase transformer working at unity power factor has an efficiency of 95% at both half load and full load of 1500W. Determine the efficiency at 60% of full load.
12. A single-phase 10KVA, 11000/220V transformer has core loss PC = 300 w at rated voltage and a copper loss P_{CU} = 400 w at full load. Find the efficiency of transformer feeding to a load 8 KVA at 0.8 p.f lagging.
13. A single phase transformer working at unity power factor has an efficiency of 95% at both half load and full load of 1500W. Determine the efficiency at 60% of full load?
14. A 5KVA, 200/100V, 1phase, 50Hz has a rated secondary voltage at full load. When the load is removed the secondary voltage is found to be 110V. Determine percentage regulation.

UNIT V
SECTION – A
Objective Questions

1. Which of the following does not change in a transformer? [ ]
   a) Voltage b)Current c)frequency d)all of the above

2. A transformer is laminated to [ ]
   a) reduce hysteresis loss b) reduce eddy current loss
   c) reduce copper loss\ d)reduce all the above losses

3. The path of magnetic flux in a transformer should have [ ]
   a) high resistance b) high reluctance
   c) low resistance d) low reluctance

4. Primary winding of a transformer [ ]
   a) is always a low voltage winding
   b) is always a high voltage winding
   c) could either be a high voltage winding or a low voltage winding
   d) none of the above

5. Which winding in a transformer has more number of turns? [ ]
   a) Primary winding b) secondary winding
   c) low voltage winding d) High voltage winding

6. A transformer [ ]
   a) changes AC to DC
   b) changes DC to AC
   c) Steps up or down DC voltages and currents
   d) steps up or down AC voltages and currents
7. The rating of a transformer is expressed in [ ]
   a) KW b)KVA c)KVAR d) none of the above
8. What happens when the primary of a transformer is given a DC supply?
   a) The transformer may start to smoke and burn [ ]
   b) No effect
   c) transformer may operate at low efficiency
   d) transformer may operate at a high effifciency

9. A step up transformer [ ]
   a) Step up the level of voltage b) Step down the level of current
   c) Step up the level of power d) step up the level of frequency
   e) only a and b

10. Open circuit is conducted to determine [ ]
    a) Hysteris losses b) Core losses
    c) Eddy current losses d) Copper losses

11. During open circuit test on a transformer [ ]
    a) Primary is supplied with rated voltage
    b) Primary is supplied full load current
    c) Primary is supplied current at reduced voltage
    d) Primary is supplied with rated KVA

12. Maximum efficiency occurs when copper loss is ___ iron loss [ ]
    a) equal to the
    b) greater than the
    c) Less than the
    d) none of the above

13. The primary and secondary windings of a transformer are [ ]
    a) Inductively coupled
    b) conductively coupled
    c) electrically coupled
    d) mechanically coupled

SECTION – B

Descriptive Questions

1. a) What is the efficiency of a transformer? Obtain the condition for maximum efficiency.

   b) List out various losses in transformer.
2. Explain the procedure for conducting open circuit and Short circuit test on transformers.
3. Define voltage regulation. Derive an expression for voltage regulation of a single phase transformer.
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13. A single phase transformer working at unity power factor has an efficiency of 95% at both half load and full load of 1500W. Determine the efficiency at 60% of full load?
14. A 5KVA, 200/100V, 1phase, 50Hz has a rated secondary voltage at full load. When the load is removed the secondary voltage is found to be 110V. Determine percentage regulation.

UNIT – VI

THREE PHASE INDUCTION MOTORS

Objectives:

- To impart some basic knowledge on three phase induction motors to students.
- To familiarize students with the applications of three phase induction motors.

Syllabus:

Principles of operation, Constructional details, slip, rotor frequency and torque equation (All the above topics are only elementary treatment and simple problems).

Learning Outcomes:

Student will be able to

- Understand the principle of operation of three phase induction motor.
- Understand the construction of three phase induction motor.
- Understand the concepts of slip and torque.

Introduction:

The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load. However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required. Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. In this chapter, we shall focus our attention on the general principles of 3-phase induction motors.

Three-Phase Induction Motor:

Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding). Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name. The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a ―transformer type□ a.c. machine in which electrical energy is converted into mechanical energy.

Advantages:

- It has simple and rugged construction.
- It is relatively cheap.
- It requires little maintenance.
- It has high efficiency and reasonably good power factor.

- It has self starting torque.

Disadvantages:

- It is essentially a constant speed motor and its speed cannot be changed easily.
- Its starting torque is inferior to d.c. shunt motor

Construction:

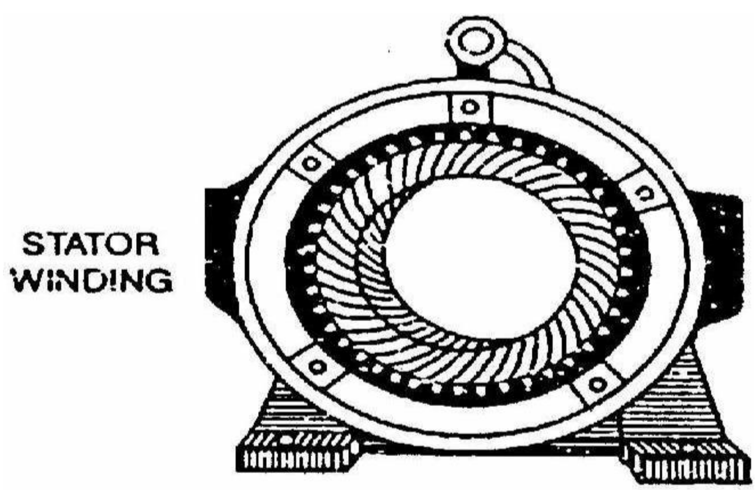
A 3-phase induction motor has two main parts

(i) stator and
(ii) rotor

The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.

Stator:

It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations [See Figure]. The insulated connected to form a balanced 3-phase star or delta connected circuit. The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field (See figure) of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.



Rotor:

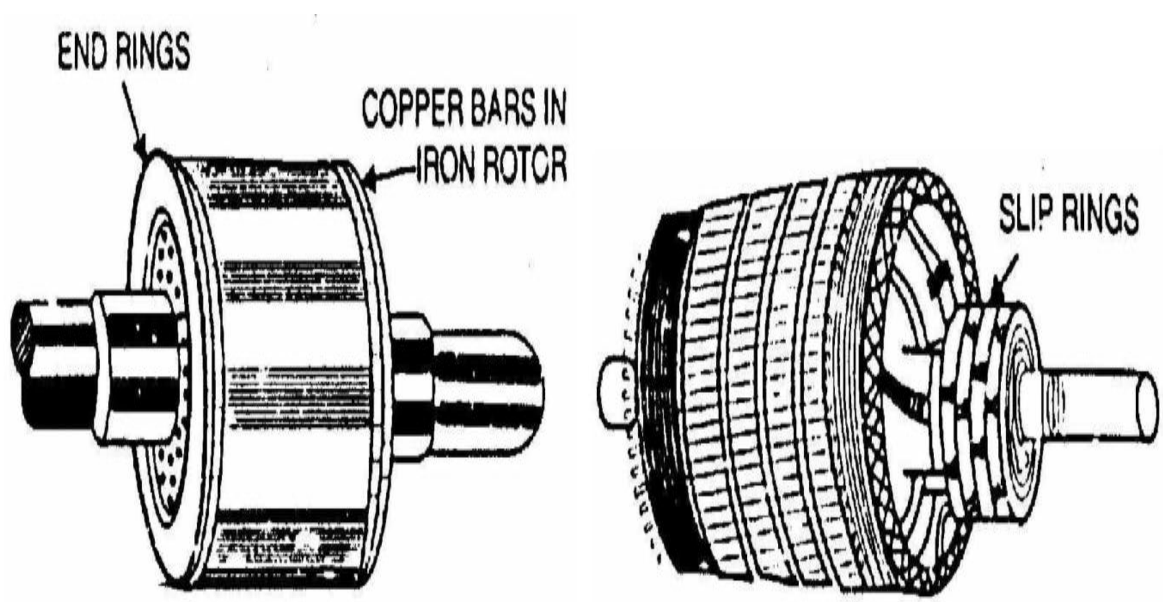
The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

(i) Squirrel cage type
(ii) Wound type

Squirrel cage rotor:

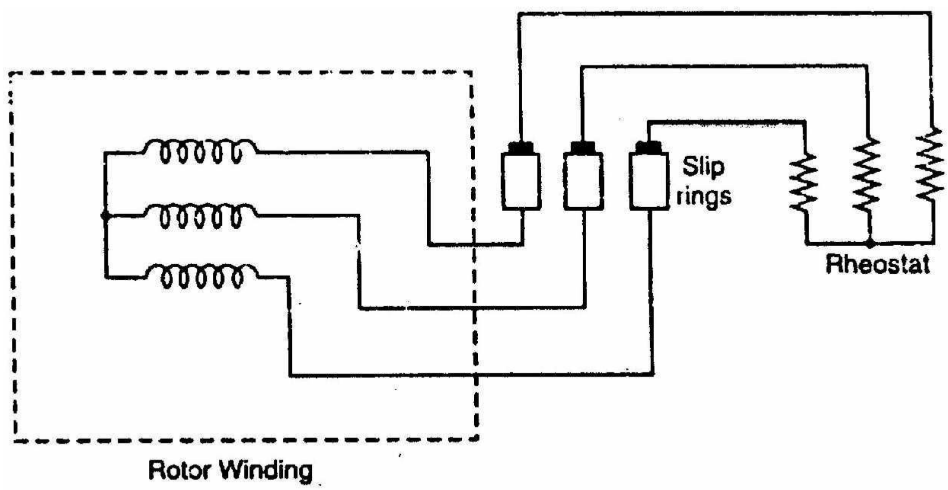
It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Figure].This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.

Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.



Wound rotor:

It consists of a laminated cylindrical core and carries a 3- phase winding; similar to the one on the stator [See Figure].The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The three brushes are connected to a 3-phase star-connected rheostat as shown in Figure at starting; the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed.



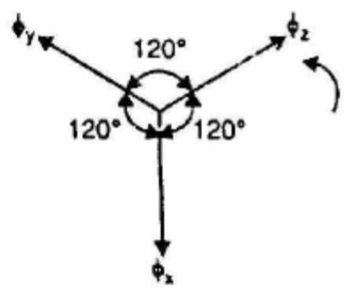
The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

Rotating Magnetic Field Due to 3-Phase Currents:

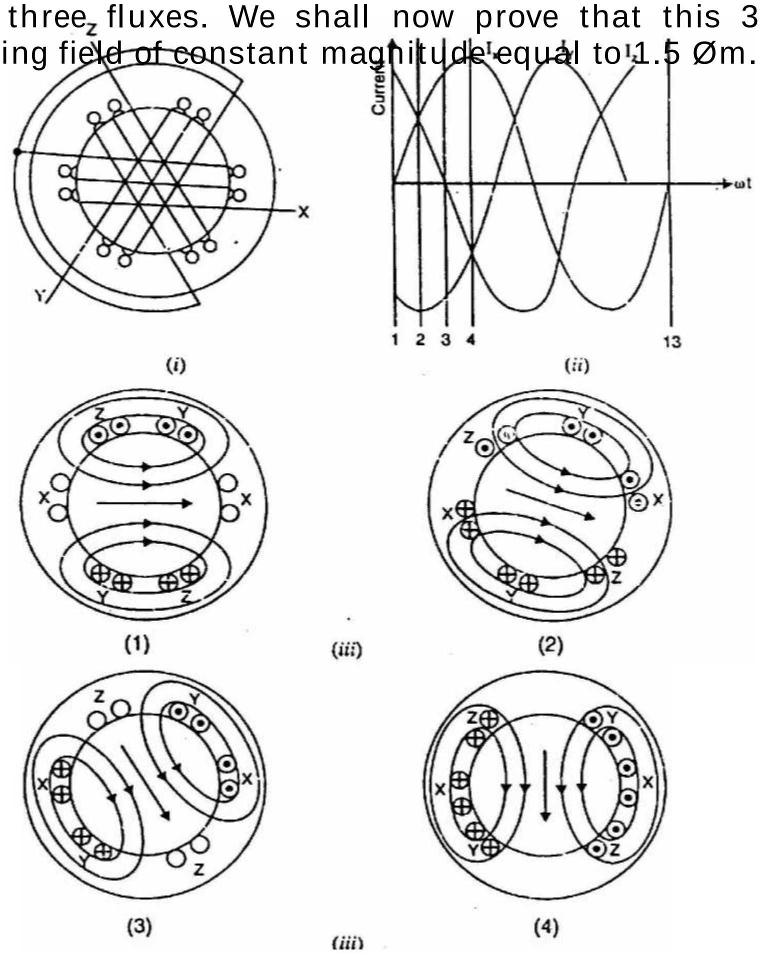
When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do no remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating Held. It can be shown that magnitude of this rotating field is constant and is equal to 1.5 Øm where Øm is the maximum flux due to any phase.

To see how rotating field is produced, consider a 2-pole, 3i-phase winding as shown in Figure The three phases X, Y and Z are energized from a 3-phase source and currents in these phases are indicated as Ix, Iy and Iz See Figure referring to Figure the fluxes produced by these currents are given by:

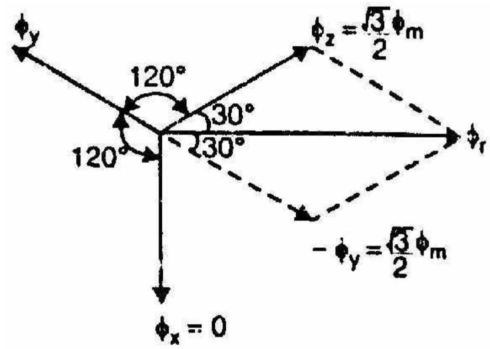
$$\phi_x = \phi_m \sin \omega t$$
$$\phi_y = \phi_m \sin (\omega t - 120°)$$
$$\phi_z = \phi_m \sin (\omega t - 240°)$$



Here Øm is the maximum flux due to any phase. Figure shows the phasor diagram of the three fluxes. We shall now prove that this 3-phase supply produces a rotating field of constant magnitude equal to 1.5 Øm.



At instant 1, the current in phase X is zero and currents in phases Y and Z are equal and opposite. The currents are flowing outward in the top conductors and inward in the bottom conductors. This establishes a resultant flux towards right. The magnitude of the resultant flux is constant and is equal to 1.5 Øm as proved under:



At instant 1, wt = 0°. Therefore, the three fluxes are given by

$$\phi_x = 0; \qquad \phi_y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m;$$

$$\phi_z = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$$

The phasor sum of - Øy and Øz is the resultant flux Ør [See Figure] it is clear that:

$$\text{Resultant flux, } \phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60°}{2} = 2 \times \frac{\sqrt{3}}{2}\phi_m \times \frac{\sqrt{3}}{2} = 1.5\ \phi_m$$

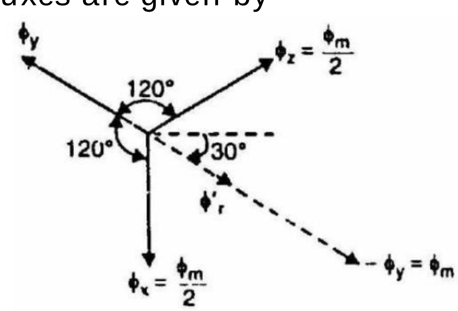
At instant 2, the current is maximum (negative) in Øy phase Y and 0.5 maximum (positive) in phases X and Y. The magnitude of resultant flux is 1.5Øm as proved under:

At instant 2, wt = 30°. Therefore, the three fluxes are given by

$$\phi_x = \phi_m \sin 30° = \frac{\phi_m}{2}$$

$$\phi_y = \phi_m \sin(-90°) = -\phi_m$$

$$\phi_z = \phi_m \sin(-210°) = \frac{\phi_m}{2}$$



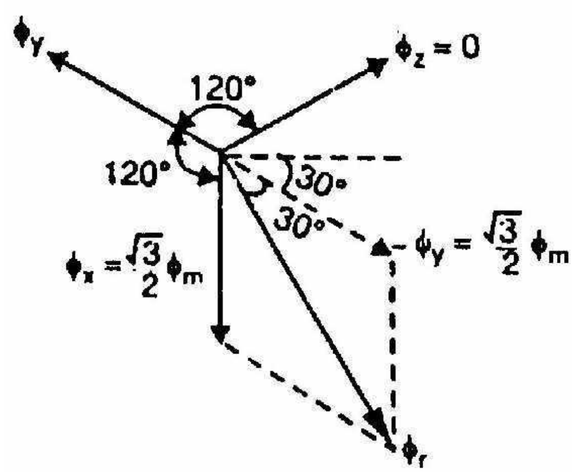
The phasor sum of ϕ_x, $-\phi_y$ and ϕ_z is the resultant flux ϕ_r

Phasor sum of ϕ_x and ϕ_z, $\phi'_r = 2 \times \frac{\phi_m}{2}\cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and $-\phi_y$, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\ \phi_m$

Note that resultant flux is displaced 30° clockwise from position 1.

At instant 3, current in phase Z is zero and the currents in phases X and Y are equal and opposite (currents in phases X and Y arc 0.866 X max. value). The magnitude of resultant flux is 1.5 Øm as proved under



At instant 3, wt = 60°. Therefore, the three fluxes are given by

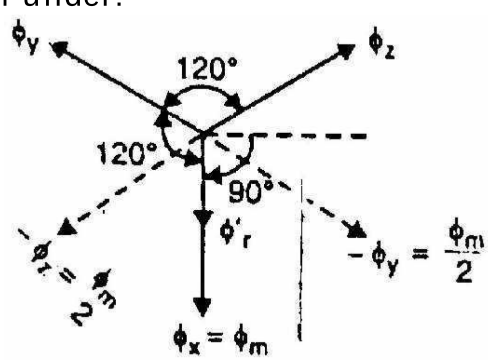
$$\phi_x = \phi_m \sin 60^\circ = \frac{\sqrt{3}}{2}\phi_m;$$
$$\phi_y = \phi_m \sin(-60^\circ) = -\frac{\sqrt{3}}{2}\phi_m;$$
$$\phi_z = \phi_m \sin(-180^\circ) = 0$$

The resultant flux Ør is the phasor sum of Ox and - Øy (Øz = 0).

$$\phi_r = 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60^\circ}{2} = 1.5\ \phi_m$$

Note that resultant flux is displaced 60° clockwise from position 1.

At instant 4, the current in phase X is maximum (positive) and the currents in phases V and Z are equal and negative (currents in phases V and Z are 0.5 X max. value). This establishes a resultant
flux downward as shown under:



At instant 4, wt = 90°. Therefore, the three fluxes are given by;

$$\phi_x = \phi_m \sin 90° = \phi_m$$

$$\phi_y = \phi_m \sin(-30°) = -\frac{\phi_m}{2}$$

$$\phi_z = \phi_m \sin(-150°) = -\frac{\phi_m}{2}$$

The phasor sum of ϕ_x, $-\phi_y$ and $-\phi_z$ is the resultant flux ϕ_r

Phasor sum of $-\phi_z$ and $-\phi_y$, $\phi'_r = 2 \times \frac{\phi_m}{2} \cos\frac{120°}{2} = \frac{\phi_m}{2}$

Phasor sum of ϕ'_r and ϕ_x, $\phi_r = \frac{\phi_m}{2} + \phi_m = 1.5\,\phi_m$

Note that the resultant flux is downward i.e., it is displaced 90° clockwise from position 1

It follows from the above discussion that a 3-phase supply produces a rotating field of constant value (= 1.5Øm, where fm is the maximum flux due to any phase).

Speed of rotating magnetic field:

The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns) Referring to Figure the time instant 4 represents the completion of one-quarter cycle of alternating current Ix from the time instant 1. During this one quarter cycle, the field has rotated through 90°. At a time instant represented by 13 or one complete cycle of current Ix from the origin, the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general, fur P poles, the rotating field makes one revolution in P/2 cycles of current.

Cycles of current = P/2 x revolutions of field

Cycles of current per second = P/2 x revolutions of field per second

Since revolutions per second is equal to the revolutions per minute (Ns) divided by 60 and the number of cycles per second is the frequency f,

$$\therefore \quad f = \frac{P}{2} \times \frac{N_s}{60} = \frac{N_s P}{120}$$

$$N_s = \frac{120\,f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

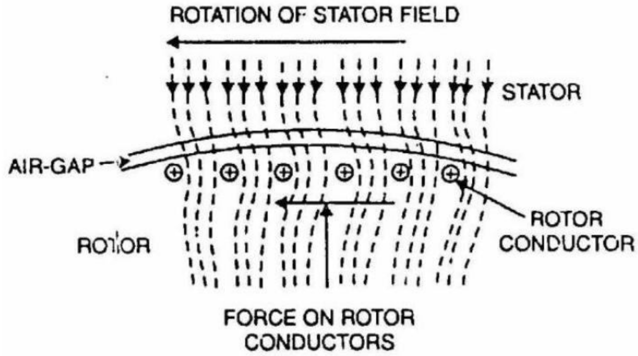
Direction of rotating magnetic field:

The phase sequence of the three-phase voltage applied to the stator winding in Figure is X-Y-Z. If this sequence is changed to X-Z-Y, it is observed that direction of rotation of the field is reversed i.e., the field rotates counterclockwise rather than clockwise. However, the number of poles and the speed at which the magnetic field rotates remain unchanged. Thus it is necessary only to change the phase sequence in order to change the direction of rotation of the magnetic field. For a three-phase supply, this can be done by interchanging any two of the three lines. As we shall see, the rotor in a 3-phase induction motor runs in the same direction as the rotating magnetic field. Therefore, the direction of rotation of a 3-phase induction motor can be reversed by interchanging any two of the three motor supply lines.

Principle of Operation:

Consider a portion of 3-phase induction motor as shown in Figure the operation of the motor can be explained as under:

- When 3-phase stator winding is energized from a 3-phase supply, a rotating magnetic field is set up which rotates round the stator at synchronous speed Ns (= 120 f/P).
- The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary. Due to the relative speed between the rotating flux and the stationary rotor, e.m.f.s are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors.
- The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field.
- The fact that rotor is urged to follow the stator field (i.e., rotor moves in the direction of stator field) can be explained by Lenz's law. According to this law, the direction of rotor currents will be such that they tend to oppose the cause producing them. Now, the cause producing the rotor currents is the relative speed between the rotating field and the stationary rotor conductors. Hence to reduce this relative speed, the rotor starts running in the same direction as that of stator field and tries to catch it.



Slip:

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suitor field speed (Ns). This difference in speed depends upon load on the motor.

The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.

$$\%\text{ age slip, } s = \frac{N_s - N}{N_s} \times 100$$

- The quantity Ns - N is sometimes called slip speed.
- When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
- In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

Rotor Current Frequency:

The frequency of a voltage or current induced due to the relative speed between a vending and a magnetic field is given by the general formula

$$\text{Frequency} = \frac{NP}{120}$$

Where N = Relative speed between magnetic field and the winding
P = Number of poles

For a rotor speed N, the relative speed between the rotating flux and the rotor is Ns - N. Consequently, the rotor current frequency f' is given by;

$$f' = \frac{(N_s - N)P}{120}$$

$$= \frac{sN_sP}{120}$$

$$= sf$$

$$\left(\because s = \frac{N_s - N}{N_s}\right)$$

$$\left(\because f = \frac{N_sP}{120}\right)$$

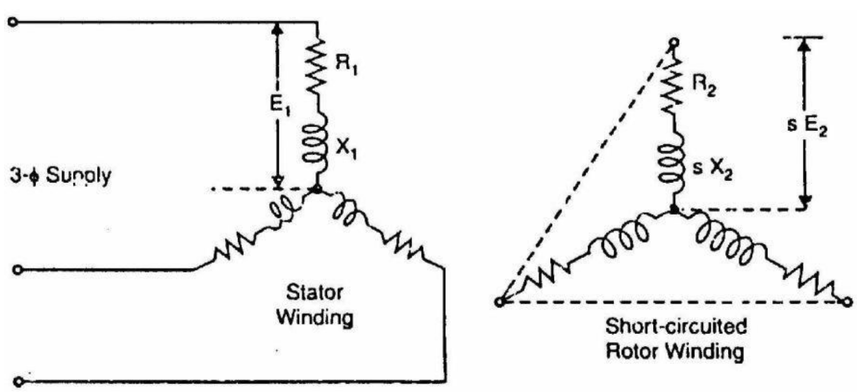
i.e., Rotor current frequency = Fractional slip x Supply frequency
(i) When the rotor is at standstill or stationary (i.e., s = 1), the frequency of rotor current is the same as that of supply frequency (f' = sf = 1x f = f).

As the rotor picks up speed, the relative speed between the rotating flux and the rotor decreases. Consequently, the slip s and hence rotor current frequency decreases.

The relative speed between the rotating field and stator winding is Ns – 0 = Ns. Therefore, the frequency of induced current or voltage in the stator winding is f = Ns P/120—the supply frequency.

Rotor Current:

Figure shows the circuit of a 3-phase induction motor at any slip s. The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are s E2 and sX2 respectively. The rotor resistance/phase is R2 and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values R1 and X1 do not depend upon slip.

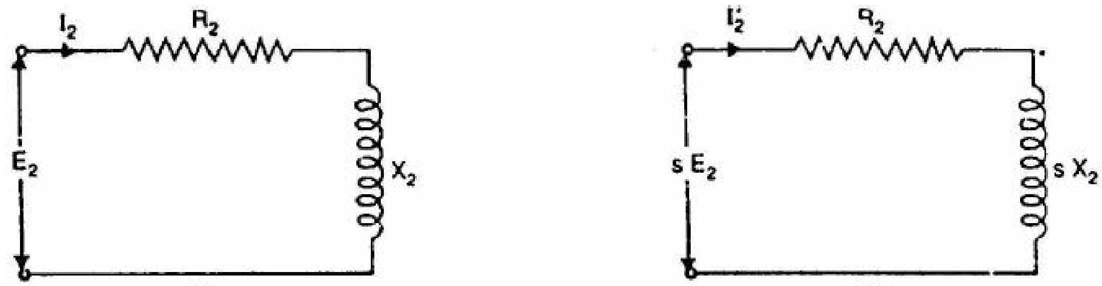


Since the motor represents a balanced 3-phase load, we need consider one phase only; the conditions in the other two phases being similar.

At standstill. Figure shows one phase of the rotor circuit at standstill.

$$\text{Rotor current/phase, } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\text{Rotor p.f., } \cos\phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$



(*i*) (*ii*)

When running at slip s. Fig. (ii) Shows one phase of the rotor circuit when the motor is running at slip s.

$$\text{Rotor current, } I'_2 = \frac{sE_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_2 = \frac{R_2}{Z'_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Rotor Torque:

The torque T developed by the rotor is directly proportional to:

- Rotor current
- Rotor e.m.f
- Power factor of the rotor circuit

$$\therefore \quad T \propto E_2 I_2 \cos\phi_2$$

$$T = K E_2 I_2 \cos\phi_2$$

$$I_2 = \text{rotor current at standstill}$$
$$E_2 = \text{rotor e.m.f. at standstill}$$
$$\cos\phi_2 = \text{rotor p.f. at standstill}$$

Note. The values of rotor e.m.f., rotor current and rotor power factor are taken for the given conditions

Torque under Running Conditions:

Let the rotor at standstill have per phase induced e.m.f. E2, reactance X2 and resistance R2. Then under running conditions at slip s,

Rotor e.m.f./phase, E'2 = sE2
Rotor reactance/phase, X'2 = sX2

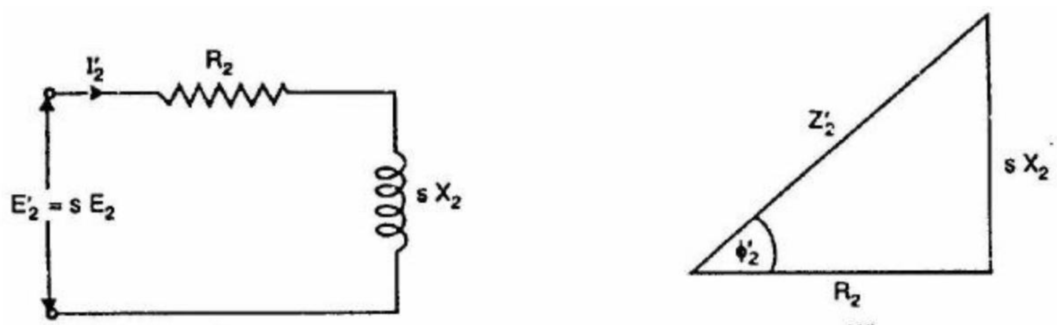
$$\text{Rotor impedance/phase, } Z'_2 = \sqrt{R_2^2 + (sX_2)^2}$$

$$\text{Rotor current/phase, } I'_2 = \frac{E'_2}{Z'_2} = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$\text{Rotor p.f., } \cos\phi'_m = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Running Torque, Tr □□E'2 I'2 cos□'2

□□□□□□□□□□I'2 cos□'2 (E'2 □□□)



(i). (ii)

$$\propto \phi \times \frac{s\,E_2}{\sqrt{R_2^2 + (s\,X_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (s\,X_2)^2}}$$

$$\propto \frac{\phi\, s\, E_2\, R_2}{R_2^2 + (s\,X_2)^2}$$

$$= \frac{K\,\phi\, s\, E_2\, R_2}{R_2^2 + (s\,X_2)^2}$$

$$= \frac{K_1\, s\, E_2^2\, R_2}{R_2^2 + (s\,X_2)^2} \qquad (\because E_2 \propto \phi)$$

If the stators supply voltage V is constant, then stator flux and hence E2 will be constant.

$$\therefore \quad T_r = \frac{K_2\, s\, R_2}{R_2^2 + (s\,X_2)^2}$$

Where K2 is another constant

It may be seen that running torque is:

- Directly proportional to slip i.e., if slip increases (i.e., motor speed decreases), the torque will increase and vice-versa.
- Directly proportional to square of supply voltage (E2 □□V)

It can be shown that value of K1 = 3/2 □□Ns where Ns is in r.p.s.

$$\therefore \quad T_r = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{R_2^2 + (s\,X_2)^2} = \frac{3}{2\pi N_s} \cdot \frac{s\,E_2^2\,R_2}{(Z'_2)^2}$$

At starting, s = 1 so that starting torque is

$$T_s = \frac{3}{2\pi N_s} \cdot \frac{E_2^2\,R_2}{R_2^2 + X_2^2}$$

UNIT – VI
SECTION – A

Objective Questions

1. The frequency of the induced emf in an induction motor is [ ]
   a. Greater than the supply frequency
   b. Lesser than the supply frequency
   c. Same as the supply frequency
   d. None of these

2. The resultant flux in an induction motor is equal to the [ ]
   a. Maximum value of flux due to any phase
   b. Twice of the maximum value of flux due to any phase
   c. 0.5 times the maximum value of flux due to any phase
   d. 1.5 times the maximum value of flux due to any phase

3. In induction motor, greater the number of poles [ ]
   a. Lesser the speed b. Greater the speed
   c. Lesser the frequency d. All of these

4. In an induction motor, rotor speed is always [ ]
   a. Less than the stator speed b. More than the stator speed
   c. Equal to the stator speed d. None of these

5. Slip of an induction motor increases with [ ]
   a. increase in current and decrease in torque
   b. increase in current and torque
   c. decrease in current and torque
   d. decrease in current and increase in torque

6. The rotor slots of the 3 phase induction motor are inclined. This phenomenon is known as: [ ]
   a) skewing b) crawling c) cogging d) none of the above

7. When the induction motor is stand still the slip will be: [ ]
   a) zero b) one c) infinity d) 0.5

8. 4. The power factor of the 3 phase induction motor will be maximum when it operates at: [ ]

a) Full load b) No-load

c) Maximum slip d) Maximum torque

9. In squirrel cage induction motors the rotor slots are usually given slight skew in order to [ ]

a) Reduce windage losses

b) Reduce eddy currents

c) Reduce accumulation of dirt and dust

d) Reduce magnetic hum

10. The efficiency of an induction motor is expected to be nearly equal to

a) 60-90% c)80-90% [ ]

b) 85-98% d) 99%

SECTION – B

Descriptive Questions

1) Derive the expression for torque of an induction motor.

2) Explain the principle of operation of 3-phase induction motor.

3) Explain Rotating Magnetic Field (RMF) concept of a 3-phase induction motor.

4) Explain the construction of a 3-phase induction motor.

5) A 4 pole, 3-phase, 50 Hz, induction motor has a star connected rotor. The rotor has a resistance of 0.1 Ω per phase and standstill reactance of 2 Ω per phase. The induced e.m.f between the slip rings is 100 V. If full load speed is 1460 rpm, find, i) Slip ii) rotor frequency iii) rotor current iv) Rotor power factor on full load condition.

6) A 3-phase, 400V, 50 Hz, 4 pole induction motor has star connected stator winding. The rotor resistance and reactance are 0.1 Ω and 1 Ω respectively. The full load speed is 1440 rpm. Calculate the torque developed on full load by the motor

7) A 6 pole induction motor is fed from 50Hz supply. If the frequency of rotor emf at full load is 2Hz, find the full load speed and slip.

8) What is rotor frequency of induction motor? A 4 pole, 3Ø induction motor operates from a supply whose frequency is 50Hz.Calculate the speed of the rotor when the slip is 0.04.

9) A star connected rotor of 3-phase, 4 pole, 50 Hz induction motor has standstill impedance of (0.35+j2) Ω per phase. When stator is given a supply, the rotor induced e.m.f is 160 V. Calculate the rotor current and rotor p.f when the machine running at 1410 r.p.m.

10) A 3-phase, 6 pole, 50 Hz induction motor has a slip of 1% at no load and 3% at full load. Find i) no load speed ii) full load speed iii) frequency of rotor current on full load.

11) The voltage applied to the stator of a 3-phase, 4 pole induction motor has a frequency of 50 Hz. The frequency of the emf induced in the rotor is 2 Hz. Calculate the slip and speed at which the motor is running.

12) What is slip of an Induction Motor? If a 6 pole 3 phase Induction Motor is connected to 50 Hz supply. It is running at 970 r.p.m. Find the slip?